The sacred-geometrical basis of the $E_8 \times E_8$ heterotic superstring symmetry group

(For more details, see “4-d sacred geometries/Polychorons & Gosset polytope” at: www.smphillips.mysite.com)
The Coxeter plane projection of the 240 vertices of the $4_{21}$ polytope.

Four triacontagons with 120 red vertices and a smaller copy of these with 120 blue vertices.

The 120 vertices of the 600-cell form four triacontagons.

The 120 vertices of a smaller 600-cell form four triacontagons.

The interior angle of a triacontagon is 168°.

Each of the half-revolutions of a helical whorl of the UPA comprises 168 circular turns.

Each quarter-revolution of a whorl of the UPA comprises 84 turns.

The Kabbalistic connection between the UPA and the $4_{21}$ polytope

The gematria number value of Cholem Yesodeth, the Mundane Chakra of Malkuth, is 168. This is both the size of the interior angle of each triacontagon in the Coxeter plane projection of the $4_{21}$ polytope and the number of turns in every half-revolution of a whorl of the UPA.
3 + 5 + 7 + ... + 81 = 1680
83 + 85 + 87 + ... + 141 = 2×1680

3 + 5 + 7 + ... + 141 = 3×1680 = 5040 =

A sector of the triacontagon

6° + 84° = 90°

Assigning the first 70 odd integers after 1 to the 70 yods in the Tree of Life generates the number 5040 as their sum.

Plato's Lambda Tetractys

Type B hexagon

The 6:84 division of angles in the two right-angled triangles making up each sector of the triacontagon conforms to the pattern of numbers in Plato's Lambda Tetractys and to the distribution of yods in the Type B hexagon.
A whorl is a helix with 1680 turns. It makes 5 revolutions around the spin axis of the UPA.

504 yods surround the centre of the heptagon.

The three major whorls of the UPA/subquark superstring have (3×1680=5040) turns spread along their (3×10=30) half-revolutions, each with 168 turns.

504 geometrical elements surround the axis of the disdyakis triacontahedron.

504 yods (3 sets of 168 yods) surround the centre of the Type C dodecagon.

Sum of interior angles of the triacontagon = 30×168° = 5040°. This factorisation is identical to that of the 5040 circular turns of the three major whorls of the UPA, each one making 10 half-revolutions of 168 turns. The 30 vertices of the triacontagon are analogous to the 30 half-revolutions of the major whorls. Each interior angle 168° is the sum of two angles of 84°.

Circles denote corners of triangles that are directly above corners of triangles in the next lower layer.

504 hexagonal yods on the 252 sides of the 126 tetractyses in the 42 Type A triangles of the Sri Yantra.

168 yods line the 126 sides of the 63 tetractyses in the 21 Type A triangles of each half of the 3-d Sri Yantra. They consist of two sets of 84 yods.

The triacontagon is the Petrie polygon of the 4_{21} polytope with E_8 symmetry.

<table>
<thead>
<tr>
<th>Corners</th>
<th>Sides</th>
<th>Triangles</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Faces</td>
<td>60 vertices</td>
<td>180</td>
<td>120×3 = 360</td>
</tr>
<tr>
<td>Subtotal</td>
<td>180</td>
<td>540</td>
<td>360</td>
</tr>
<tr>
<td>Interior</td>
<td>180</td>
<td>60</td>
<td>180×3 = 540</td>
</tr>
<tr>
<td>Subtotal</td>
<td>180</td>
<td>600</td>
<td>540</td>
</tr>
<tr>
<td>Interior</td>
<td>360</td>
<td>120</td>
<td>360×3 = 1080</td>
</tr>
<tr>
<td>Subtotal</td>
<td>360</td>
<td>1200</td>
<td>1080</td>
</tr>
<tr>
<td>Total</td>
<td>720</td>
<td>2340</td>
<td>1180</td>
</tr>
</tbody>
</table>
The (7+7) enfolded regular polygons of the inner form of the Tree of Life.

(180+180=360) yods surround the centres of the two Type B dodecagons. They may be interpreted as symbolising the 360 degrees of a full rotation.

The 180 yods surrounding the centre of the Type B dodecagon consist of its 12 corners and 168 yods. Six sectors have 84 black yods and six sectors have 84 white yods.

The 168 yods surrounding the centre of the Type B dodecagon that are not its corners consist of 60 red yods making up the Type A dodecagon and 108 blue yods. This division corresponds to the fact that the triacontagon is the largest regular polygon whose interior angle (168°) is the sum of the interior angles of smaller polygons: 168° = 60° (triangle) + 108° (pentagon).

The triacontagon has 30 corners. Its interior angle is 168°. The two base angles of an isosceles sector are 84° and the angle at its apex is 12°. The fact that the interior angle is (apart from a factor of 10) the same number that Charles Leadbeater counted in the turns of a helical whorl is powerful evidence that the UPA is an $E_8 \times E_8$ heterotic superstring because it is implausible that a number with such paranormal provenance could appear by chance in the basic geometry of a symmetry group that is associated with superstrings.

As the Petrie polygon of the $4_{21}$ polytope, the triacontagon conforms to the holistic 12:84:84 pattern shown by the Type B dodecagon and the disdyakis triacontahedron.
Number of yods in the Type B n-gon = 15n + 1.
Number of yods surrounding the centres of 7 Type B polygons with 48 corners = 15×48 = 720.
The composition of yods is:

- (4+3) polygons
- (24+24) polygons
- 168° + 168°
- 168° + 168°

Total = 360 + 360 = 720

**Correspondences**

- 48 corners of each set of 7 polygons → sum (48°) of vertex angles of one sector in each of the 4 triacontagons making up each 600-cell.
- 4 sets of 168 yods in each set of 7 polygons → sum (4×168) of base angles of a sector in each of the 4 triacontagons.
- 720 yods in each set of 7 polygons → sum (720°) of angles in a sector of each of the 4 triacontagons in each 600-cell.
Each helical whorl of the UPA has 1680 circular turns (168 turns in each half-revolution).

The outer form of 10 Trees of Life comprises 1680 yods. The UPA (subquark state of the $E_8 \times E_8$ heterotic superstring) is its microscopic manifestation. Each yod denotes a turn of the whorl. ADONAI, the Godname of Malkuth, prescribes this structural parameter because its number value 65 is the number of Sephirothic emanations below the top of the 10th Tree.

The inner form of 10 Trees of Life. The $4_{21}$ polytope representing the 240 roots of $E_8$ is its microscopic manifestation. Each side of a triangle corresponds to an edge of the $4_{21}$ polytope.

Coxeter plane projection of the 240 vertices of the $4_{21}$ polytope with 6720 edges. It contains eight circular rings (triacontagons with 30 vertices).

The Kabbalistic connection between the UPA and the $4_{21}$ polytope.

A Type C polygon has 14 sides of 9 triangles per sector. The 7 separate polygons of the inner Tree of Life have 48 sectors. The 7 separate, Type C polygons have $(48 \times 9 = 432)$ triangles with $(48 \times 14 = 672)$ sides.

The outer form of 10 Trees of Life comprises 1680 yods. The UPA (subquark state of the $E_8 \times E_8$ heterotic superstring) is its microscopic manifestation. Each yod denotes a turn of the whorl. ADONAI, the Godname of Malkuth, prescribes this structural parameter because its number value 65 is the number of Sephirothic emanations below the top of the 10th Tree.
Interior angle of triacontagon = 168°.
Sum of 30 interior angles = 30×168° = 5040° = 71°
4-d Coxeter plane projection of the 421 polytope consists of 4 triacontagons with 120 red vertices and 4 smaller triacontagons with 120 blue vertices.
Sum of 240 interior angles of 8 triacontagons = 8×71° = 8!° = 40320°.

The inner form of the Tree of Life consists of 4 red polygons with 24 sectors and 3 light blue polygons with 24 sectors. 21 yods and 21 yods in each sector surround the centre of each red polygon. 21 yods and 21 yods in each sector surround the centre of each light blue polygon. The 4 red polygons have (24×21=504) & 504 yods; the 3 light blue polygons have 504 yods & 504 yods. The (40+30=70) polygons have 4 sets of 5040 (=7!) yods. The (70+70) polygons have 2×4 sets of 7! yods, i.e., 8! yods. This is the sum in degrees of the 240 interior angles of the 8 triacontagons. Each yod denotes a degree.
The 4 sets of 7! yods in the 70 polygons making up one half of the inner form of 10 Trees of Life correspond to the 4 red triacontagons with 7!° as the sum of their interior angles; the 4 sets of 7! yods in the 70 polygons that make up their mirror-image half correspond to the 4 blue triacontagons. The two sets of yods in a polygon and the two sets of polygons with 24 vertices create the counterpart of (2×2=4) triacontagons. Each half of the inner form of 10 Trees corresponds to 4 triacontagons.
The $4_{21}$ polytope has 6720 edges. The 4 rings of 120 red dots are the 120 vertices of 4 triacontagons. Similarly for the 4 rings of blue dots. The triacontagon is the Petrie polygon of the 600-cell.

A Type C polygon has 9 triangles with 14 sides per Type B, triangular sector.

7 separate polygons have 48 sectors.

The 7 separate Type C polygons have (9×48=432) triangles with (48×14=672) sides.

The (70+70) Type C polygons in the inner form of 10 Trees of Life have (6720+6720) sides of triangles that correspond to the (6720+6720) edges of two $4_{21}$ polytopes representing the (240+240) vertices of $E_8 \times E_8$. 

Total = 6720 sides
180 yods surround the centre of a Type A triacontagon. $4 \times 180 = 720$ yods surround the centres of the 4 Type triacontagons (red, blue, green & violet yods) in the Coxeter plane projection of a 600-cell. $2 \times 720 = 1440$ yods surround the centres of the $2 \times 4$ triacontagons in the Coxeter plane projection of a compound of two 600-cells whose 240 vertices are the vertices of the Coxeter plane projection of the 240 vertices of the $4_{21}$ polytope. Each half of the inner Tree of Life with 720 yods in 7 Type B polygons denotes the four Type A triacontagons in a 600-cell with 720 yods surrounding their centres. The complete inner form of the Tree of Life expresses the projection of the $4_{21}$ polytope in the $E_8$ Coxeter plane.
The UPA is the ground state of the subquark $E_8 \times E_8$ heterotic superstring. It consists of 10 helical whorls, each of which spirals 5 times around its axis (2½ times in its outer half, 2½ times in its narrower, inner half).

A whorl comprises 1680 circular turns (336 turns per revolution, 168 turns per half-revolution). Each of the 5 revolutions of all 10 whorls contains 3360 turns (1680 per half-revolution).

The 10 whorls of the UPA “carry” the 240 gauge charges associated with the 240 roots of $E_8$. Each of the 240 vertices of the 8 triacontagons that are the Petrie polygons of the 240 vertices of the $4_{21}$ polytope denotes a root. When the triacontagons are Type B, the number of yods other than vertices that surround their 240 sectors = 3360 (1680 in the 4 red triacontagons & 1680 in the 4 blue triacontagons). The 120 red vertices are the Coxeter plane projection of the 120 vertices of a 600-cell; the 120 blue vertices are the Coxeter plane projections of the 120 vertices of a smaller, concentric 600-cell. The 1680 yods in the 4 red Type B triacontagons symbolise the 1680 turns in an outer half-revolution of the 10 whorls. The 1680 yods in the 4 blue Type B triacontagons symbolise the 1680 turns in an inner half-revolution of the 10 whorls. The two 600-cells determine the outer and inner halves of the UPA. The 4:4 division of the 8 triacontagons exhibits the basic $168:168$ division that is characteristic of holistic systems. The number 168 is the number of degrees in the interior angle of the triacontagon!

Number of yods surrounding the centre of the Type B n-gon = 15n.

Surrounding the centres of the 8 Type B triacontagons are:

4 red triacontagons
(4×30=120) vertices
(4×30×14=1680) yods

4 blue triacontagons
(4×30=120) vertices
(4×30×14=1680) yods

(1680+1680=3360) yods surround the centres of the 8 triacontagons

The (7+7) enfolded, Type A polygons have 349 corners, sides & triangles. Number of geometrical elements in the $(7n+7n)$ polygons enfolded in n Trees of Life = 347n + 2. Number of geometrical elements outside the n root edges of these polygons = 344n + 2. The 6 green corners & 4 green sides in the two side pillars of a single Tree of Life are shared with its inner form. Number of shared geometrical elements in n Trees of Life = 8n + 2. Number of geometrical elements in the inner form of n Trees that are unshared with its outer form = 336n. The inner form of 10 Trees has 3360 unshared geometrical elements (1680 in each set of 70 polygons).
A Type B triangle has 7 corners, 15 sides & 9 triangles, i.e., 31 geometrical elements. 31 is the number value of EL ("God"), the Godname of Chesed.

A triacontagon has 30 sectors with 31 corners.

The 8 concentric triacontagons are the Petrie polygons of the $4_{21}$ polytope. Their 240 vertices consist of the 120 red vertices of a 600-cell and the 120 blue vertices of a smaller 600-cell. Surrounding the centre of these Type C triacontagons are 240 sectors comprising (240×28=6720) geometrical elements. The 120 Type B sectors of the 4 red triacontagons contain 3360 geometrical elements; they correspond to the 3360 sides of triangles in the 40 red polygons in the inner form of 10 Trees of Life. The 120 Type B sectors of the 4 blue triacontagons also contain 3360 geometrical elements; they correspond to the 3360 sides of triangles in the 30 blue polygons.

The $4_{21}$ polytope has 6720 edges.

The $4_{21}$ polytope with the symmetry of $E_8$ has as many edges (6720) as there are geometrical elements in the 8 Type C triacontagons and sides of triangles in the 70 Type C polygons making up half of the inner form of 10 Trees of Life. The 70 polygons in the other half correspond to the second $E_8$ group in $E_8 \times E_8$.
E₈ Coxeter plane projection of the 240 vertices of the 4₂₁ polytope. They form the 240 corners of \((4+4=8)\) concentric triacontagons.

Each of the 10 helical whorls of the UPA revolves 5 times around its axis and contains 1680 circular turns.

The interior angle of a triacontagon is 168°. Each base angle of an isosceles sector is 84°.

Each ¼-revolution of a whorl contains 84 circular turns.
The angles in a pair of sectors of the triacontagon and its reflection conform to the 24:168:24:168 pattern of the 192 lines & 192 broken lines of the 64 hexagrams.
4 triacontagons are the projection in the $H_4$ Coxeter plane of the 120 vertices of the 600-cell. (4×420=1680) yods other than vertices surround their shared centre when they are Type B.

8 triacontagons are the projection in the $E_8$ Coxeter plane of the 240 vertices of the $4_{21}$ polytope. When Type B, (1680+1680=3360) yods other than vertices surround their shared centre.

A half-revolution of the 10 helical whorls of the UPA comprises 1680 turns. One revolution comprises 3360 turns. The 4 Type B triacontagons in each 600-cell correspond to an outer/inner half-revolution of the 10 whorls. The 1680 turns in a single whorl are circularly polarised waves composed of 3360 plane-polarised oscillations. The pattern of division of the whole also applies to its parts because they are wholes in themselves.
4 triacontagons have:
Type A: 360 corners & sides of 120 sectors;
Type B: 840 additional geometrical elements.

4 Type B triacontagons have 840 corners & sides and 360 triangles.

8 triacontagons have:
Type A: 720 corners & sides of 240 sectors.
Type B: (840+840=1680) additional geometrical elements.

8 Type B triacontagons have (840+840=1680) corners & sides and (360+360=720) triangles, i.e., 2400 geometrical elements.

2400 geometrical elements in 240 Type B sectors = 240×(3+7)
= 720 (Type A) + 1680 (additional)

The Type A/Type B distinction generates the 3:7 pattern of the tetractys and the 720:1680 division in triacontagons that is characteristic of holistic systems. It manifests in the 240 roots of $E_8$ as the 72 roots of $E_8$ and the remaining 168 roots.
The (10+10) dodecagons enfolded in 10 Trees of Life with 2nd-order tetractyses as their sectors contain (8400+8400=16800) yods outside their root edges that surround the 240 centres of sectors denoting the 240 gauge charges of $E_8$. 
When it is a sector of a triacontagon, the 2nd-order tetractys has:

- 85 yods;
- 15 corners;
- 13 yods on each side;
- 70 hexagonal yods;
- 84 yods surrounding the centre (●);
- (85−13=72) yods per sector;
- (72−2=70) yods per sector surrounding its centre that do not coincide with corners of the triacontagon/vertices of the $4_{21}$ polytope.

The number of yods surrounding the centre of an n-gon with 2nd-order tetractyses as sectors = 72n. If the 240 sectors of the 8 concentric triacontagons are 2nd-order tetractyses, their (10×240=2400) 1st-order tetractyses have (240×72=17280) yods (2400 corners & 14880 hexagonal yods). 240 of these yods coincide with vertices of the $4_{21}$ polytope and 240 are centres of sectors, leaving 16800 yods (2100 yods per triacontagon). They denote the 16800 circular turns in the 10 whorls of the UPA, each whorl making 5 complete revolutions (10 ½-revolutions) around its axis.

8400 turns in 5 inner ½-revolutions
8400 turns in 5 outer ½-revolutions

UPA

8 concentric triacontagons in the $E_8$ Coxeter plane projection of the $4_{21}$ polytope

The geometry of the $E_8$ Coxeter plane projection of the $4_{21}$ polytope with its 8 triacontagons constructed from 2nd-order tetractyses generates the factorisations 16800 = 8×30×70 = 8×3×7×10×10. Where do the two factors of 10 denoting the 10 whorls and the 10 half-revolutions per whorl originate? As the factors 8, 3 & 7 do not contain the factor 5 (a divisor of 10), this factor must come from either 30 or 70. Although the 2nd-order tetractys comprises 10 1st-order tetractyses, the 70 yods are not evenly distributed among them, so that they do not (and cannot) consist of 10 similar sets of 7 yods. Therefore, the factor of 10 that is responsible for the 10 ½-revolutions must arise solely from the 10-fold symmetry of each triacontagon, the latter’s 5-fold symmetry generating the 5 revolutions of the 10 whorls. The second factor of 10, which is responsible for the UPA having 10 whorls, must be attributed to the 10 1st-order tetractyses that make up each sector of a triacontagon.

The 16800 turns in the whorls of the UPA are not the result of symmetry-breaking of $E_8$. They are the string manifestation of all its 240 gauge charges.