ARTICLE 67

The Tetrad (4) and its Symbol the Square Express the Parameters 168, 248, 336, 496, 672, 840 & 1680 of the $E_8 \times E_8'$ Heterotic Superstring

by

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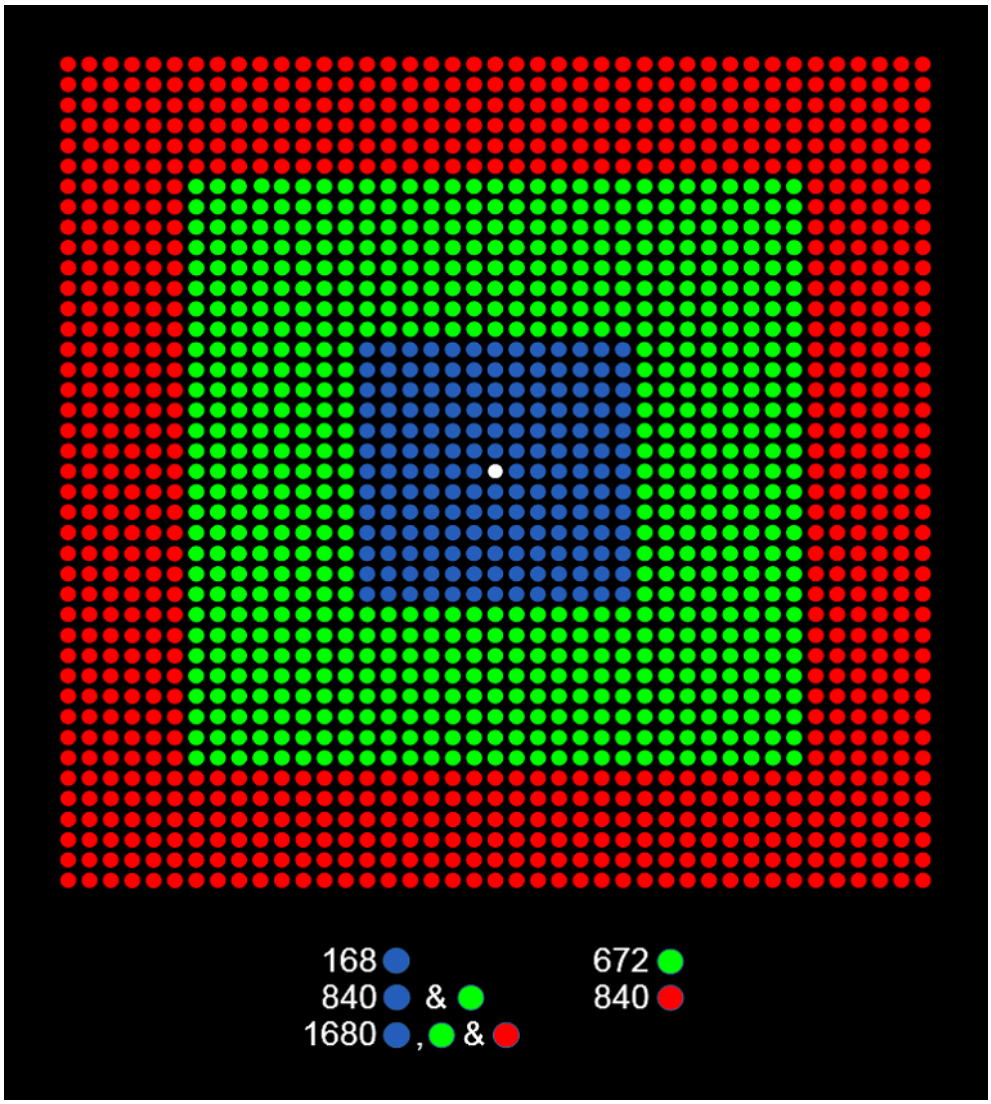
Abstract

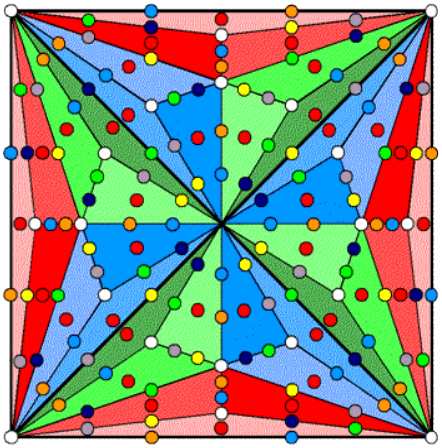
The numbers 168, 336, 672, 840 & 1680 are structural parameters of the UPA, which the Theosophists Annie Besant and C.W. Leadbeater claimed in 1895 are the basic constituent of atoms that they remote-viewed with the aid of a yogic siddhi. These numbers can be represented by concentric, square arrays of 13, 29 & 41 dots, each dot denoting the number 1. Construction of the first four Platonic solids from triangles generates these numbers as the numbers of geometrical elements that surround their axes. That this cannot, plausibly, be due to chance is supported by the fact that, when regarded as Type A polygons, the faces of the first four Platonic solids are composed of 248 points & lines, where 248 is the dimension of E_8, the rank-8, exceptional Lie group present in $E_8 \times E_8'$ heterotic superstring theory. Furthermore, for all five Platonic solids, 496 geometrical elements other than vertices on average surround an axis, where 496 is the dimension of $E_8 \times E_8'$. This is a geometrical connection provided by the Platonic solids between two group-theoretical parameters intrinsic to $E_8 \times E_8'$ heterotic superstring theory and paranormally-derived, structural parameters of the UPA. It is evidence for the latter being a state of the $E_8 \times E_8'$ heterotic superstring (according to the author's previous researches, the UPA is the basic constituent of up and down quarks). The Tetrad expresses these parameters arithmetically, whilst the square (symbol of the Tetrad) represents them geometrically. These parameters illustrate par excellence the Tetrad Principle that has been proposed by the author.

Figure A



Square representation of structural parameters of the UPA/subquark state of the $E_8 \times E_8'$ heterotic superstring.

Figure C



168 yods surround the centre of the Type C square.

Figure D

3 5 7 9

25 11

23 13

21 19 17 15

The sum of the 12 odd integers after 1 that line the sides of a square is 168.

The n×n square array of n^2 yods* has (n^2-1) yods surrounding its central yod when n is an odd integer (when n is even, no yod is at the centre of the array, so discussion will focus on odd values of n), for which there is such a yod. For n = 13, ($13^2-1=168$) yods (coloured blue in Fig. A) surround the centre. This is the gematria number value of *Cholem Yesodoth*, the Mundane Chakra of Malkuth, which is the last of the 10 Sephiroth of the Tree of Life (see here). The UPA ("Ultimate Physical Atom") is the basic unit of atomic matter described by Annie Besant & C.W. Leadbeater with *anima*, one of the siddhis of Indian yoga. It comprises 10 helical whorls (Fig. B). Each whorl has 1680 circular turns and winds five times around the axis of spin of the UPA. Its outer coil with 840 turns spirals 2½ times around this axis and returns to its starting point via a narrower spiral with 840 turns, winding another 2½ times through the core of the UPA. Each revolution comprises 336 turns and each half-revolution consists of 168 turns. Each blue yod denotes one turn as a circularly polarized oscillation created by the superposition of two orthogonal plane waves that oscillate 90° out of phase.

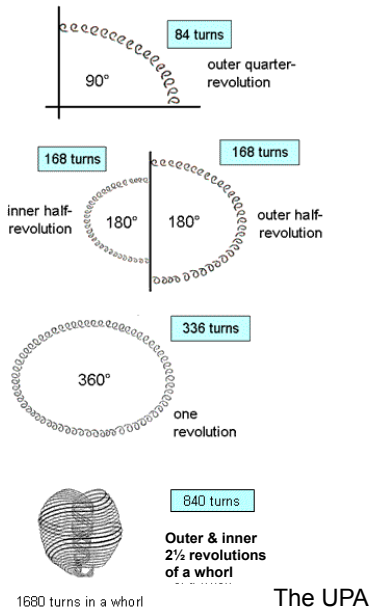


Figure B

For n = 29, the central yod of the 29×29 square array of yods in Fig. A is surrounded by ($29^2-1=840$) yods. Outside the array of 168 blue yods are ($840-168=672$) green yods surrounding its centre. The 840 green & blue yods denote either an inner or an outer half of a helical whorl, each with 840 turns.

For n = 41, ($41^2-1=1680$) yods surround the central yod in the 41×41 square array. The 840 red yods outside the 29×29 array denote the 840 turns making up either an outer or an inner half of a whorl. The two sets of 840 yods denote the two halves of each whorl of the UPA. It is not necessary here to choose which set corresponds to which half, although it seems natural to remain consistent with the analogy by regarding the two inner squares with 840 green and blue yods as symbolising the inner half of the UPA.

The number 29 is the 10th prime number and the 15th odd integer, where 15 is the number value of YAH (יה), the Godname assigned to Chokmah, which is the second member of the Supernal Triad of the Tree of Life. The number 41 is the 21st odd integer, where 21 is the number of EHYEH (אהיה), the Godname that is assigned to Kether, the first member of the Supernal Triad. This is the arithmetic way in which EHYEH prescribes each whorl of the UPA as the microphysical manifestation of the Tree of Life blueprint, its 10 whorls corresponding to the 10 Sephiroth of the latter.

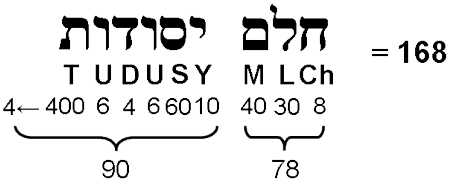
The Type A n-gon (see also here) has n sectors that are tetractyses; it has (6n+1) yods. The Type B n-gon has sectors made up of three tetractyses, i.e., they are Type A triangles; it has (15n+1) yods. The Type C n-gon has Type B triangles as its sectors; it has (42n+1) yods. For the square (n=4), 168 yods surround its centre (Fig. C).

As

$$n^2 - 1 = 3 + 5 + 7 + \ldots (2n-1),$$

$$13^2 - 1 = 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 = 168.$$

The number 168 is the sum of the first 12 odd integers after 1. When the square is constructed from tetractyses, Type A triangles, Type B triangles, etc., there are always four yods spaced evenly along each of its four sides (this is true, of course, for *any* polygon). This means that 12 yods line its sides, creating its shape. The square is an ancient symbol of the four Elements Fire, Air, Water & Earth that the ancient Greeks believed were the ingredients of matter. They are associated with Malkuth, the Sephirah that denotes the material form of anything designed according to this blueprint. It is therefore appropriate that the gematria number value of this Sephirah is the sum of the first 12 odd integers that can form the boundary of the geometric symbol of the four Elements. The sum of the first six odd integers (coloured blue in Fig. D) that are four units apart is 78, the sum of the remaining six red integers being 90. These numbers are the number values of the Hebrew words *Cholem* and *Yesodoth* making up the Kabbalistic name of the Mundane Chakra of Malkuth:



As

$$84 = 4^1 + 4^2 + 4^3$$

and

* The tenth letter of the Hebrew alphabet is yod (י). Shaped somewhat like a dot, the author uses this word to denote each point or dot in the Pythagorean tetractys symbolising the number 10, as well as in any other array of points, such as those discussed in this article.

$$336 = 4\times84 = 4^2 + 4^3 + 4^4,$$

the Tetrad (4) expresses the number of turns in each revolution of every helical whorl of the UPA (see Fig. B). Alternatively, as

$$84 = 1^2 + 3^2 + 5^2 + 7^2,$$

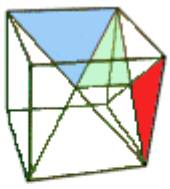
336 is the sum of the squares of the first *four* even integers that are spaced *four* units apart:

$$336 = 4\times84 = 2^2\times(1^2+3^2+5^2+7^2) = 2^2 + 6^2 + 10^2 + 14^2.$$

This illustrates the author's Tetrad Principle, discussed in his Article 1.

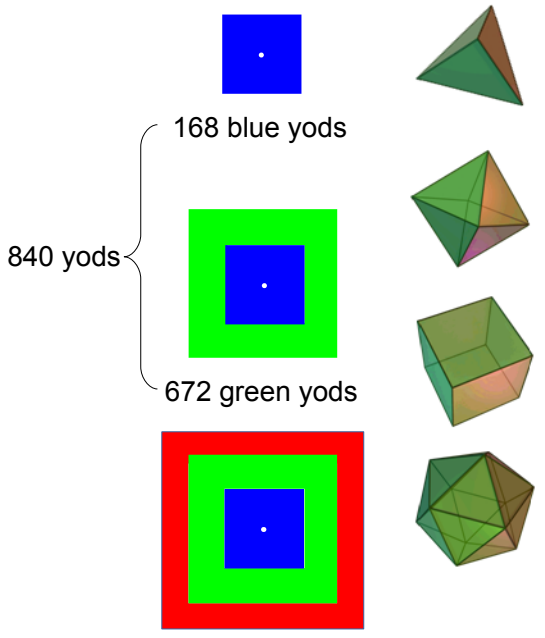
Analogy with the first four Platonic solids

The ancient Greeks thought that the particles of the four physical Elements have the shapes of the first four Platonic solids: the tetrahedron, octahedron, cube & icosahedron. The tetrahedron with four vertices and six faces was the shape of the particles of Fire, the octahedron with six vertices and eight faces was the shape of the particles of Air, the cube with eight vertices and six faces was the shape of particles of Earth and the icosahedron with twelve vertices and twenty faces was the shape of particles of Air. The dodecahedron, the fifth and last Platonic solid with twenty vertices and twelve faces, came to be associated with Aether, the fifth Element. When the centre of a Platonic sold is joined by straight lines to its vertices and to the centres of its faces, this creates internal triangles having as one side either an edge or a side of a sector of its faces. The cube shown in Fig. E is an example of this construction. There are three types of triangles, the red and blue ones having one side that is an edge of the cube and the green triangles having one side that is the side of a sector of a face. The table shown below in Figure F lists the number of points, lines & triangles that surround the axes of the first four Platonic solids when their faces are Type A polygons and all internal triangles formed by joining their centres to their vertices and centres of faces are Type A:



Construction of the faces & interior of the cube from red, green & blue triangles

Figure E



	Faces & interior triangles created by edges	Interior triangles created by sides of sectors	Total
Tetrahedron	84	84	**168**
Octahedron	**168**	**168**	336
Cube	**168**	**168**	336
Icosahedron	420	420	840
Total =	840	840	1680

672

840

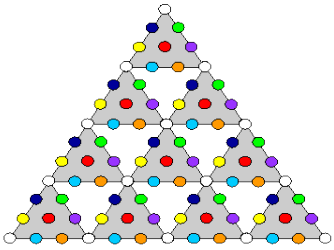
Figure F. Analogy between the three squares and the first four Platonic solids.

Surrounding its axis:

- the tetrahedron has 168 geometrical elements (84 in faces & interior triangles generated by edges and 84 in interior triangles generated by sides of sectors of faces);
- the octahedron has 336 geometrical elements (168 in faces & interior triangles generated by edges and 168 in interior triangles generated by sides of sectors of faces);
- the cube has 336 geometrical elements (168 in faces & interior triangles generated by edges and 168 in interior triangles generated by sides of sectors of faces);
- the icosahedron has 840 geometrical elements (420 in faces & interior triangles generated by edges

and 420 in interior triangles generated by sides of sectors of faces);
- all four Platonic solids have 1680 geometrical elements (840 in faces & interior triangles generated by edges and 840 in interior triangles generated by sides of sectors of faces).

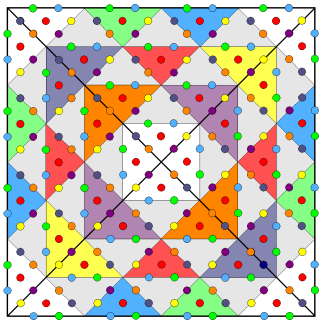
A sceptic towards the paranormal may argue that it could be just coincidence that three numbers (168, 840 & 1680 pertaining to alleged remote-viewing of subatomic particles are all of the form $n^2 - 1$, where n is an odd integer integer, in which case their representation by square arrays of dots lacks significance. This point of view is rendered implausible by the appearance of the very *same* set of numbers in the geometrical composition of the four Platonic solids that the ancient Greeks believed were the shapes of the particles of the four physical Elements. The sceptic cannot, reasonably, contend that miraculous chance can account for how exactly the *same* sequence of numbers is displayed by the geometrical composition of these Platonic solids. Consideration of the geometrical composition of the faces of the first four Platonic solids provides further refutation of this argument. The 120 triangular sectors of the 38 faces of the first four Platonic solids consist of 248 points & lines (see Table 3, p. 4 of Article 55). This is the dimension of E_8, the rank-8 exceptional Lie group present in the theory of $E_8 \times E_8'$ heterotic superstrings. In other words, the first four Platonic solids embody not only the predicted structural parameters of the subquark state of $E_8 \times E_8'$ heterotic superstrings but also the dynamical parameter 248 — the dimension of E_8! Finally, as the dual of the icosahedron, the dodecahedron is shown in Article 55 to have the same number (840) of points, lines & triangles surrounding an axis drawn through two diametrically opposite vertices. (1680+840=2520) geometrical elements surround the axes of the five Platonic solids, which have 50 vertices, 40 surrounding their axes. Therefore, (2520−40=2480) geometrical elements other than vertices surround their axes. On *average*, (2480/5=496) geometrical elements other than vertices



85 yods
15 corners (○)
70 hexagonal yods

The 2nd-order tetractys

248 =



(For the sake of clarity, yods at corners of tetractyses are not shown)

Figure G. Constructed from 2nd-order tetractyses, the square contains 248 hexagonal yods that symbolize the 248 roots of E_8.

surround the axis of a Platonic solid, each half having 248 such geometrical elements (for more details, see Article 55). The number 496 is the dimension of $E_8 \times E_8'$: 496 = 248 + 248. The average Platonic solid embodies in its geometry the very number 496 that, as physicists Michael Green and John Schwarz discovered in 1984,† is the dimension of the Yang-Mills gauge symmetry group guaranteeing interactions between 10-d superstrings

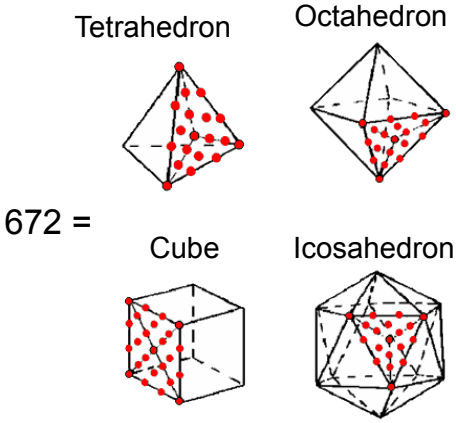


Figure H. There are 672 yods in the first four Platonic solids constructed from tetractyses.

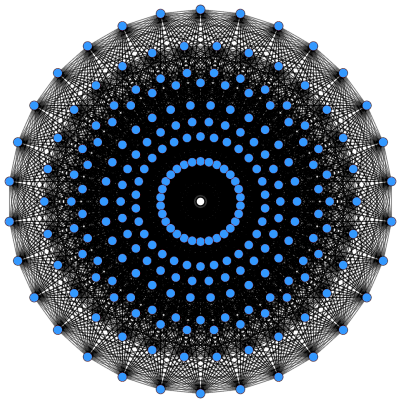


Figure I. The 240 vertices of the 4_{21} polytope are denoted by blue dots. The black lines joining them represent its 6720 edges.

that are free of quantum anomalies. The Platonic solids embody *both* theoretically-derived dynamical parameters and paranormally-derived structural parameters of the $E_8 \times E_8'$ heterotic superstrings.

† Green, M. B.; Schwarz, J. H. (1984). "Anomaly cancellations in supersymmetric D = 10 gauge theory and superstring theory". Physics Letters B. **149** (1–3): 117–122.

Square representation of the dimension 248 of E_8

The tetractys array of 10 yods made famous by Pythagoras is but the 1st-order member of an infinite class of higher-order tetractyses, starting with the mathematical point as the 0th-order tetractys. The (n+1)th-order tetractys is generated from the nth-order tetractys by replacing each of its yods by a 1st-order tetractys. The 2nd-order tetractys formed from 10 1st-order tetractyses contains 85 yods (Fig. G). This number is expressed by the Tetrad because it is the sum of the first *four* powers of 4, starting with 0:

$$85 = 4^0 + 4^1 + 4^2 + 4^3.$$

These yods comprise 15 corners (denoted in Fig. G by white circles) and 70 hexagonal yods (shown with the seven colours of the rainbow). Eight hexagonal yods line each side of the 2nd-order tetractys. When it is a sector of a polygon, they become shared with the adjacent sector, so that there are (70−8=62) hexagonal yods per sector. The four sectors of a square contain (4×62=248) hexagonal yods that symbolise the 248 roots of the Lie group E_8 appearing in $E_8 \times E_8'$ heterotic superstring theory. Notice that the 40 tetractyses have 41 corners (○). This compares with the 41 yods lining each side of the square that embodies the structural parameter 1680 of every whorl of the UPA.

Assigning the Tetrad (4) to yods in the square arrays

Suppose that the Tetrad is assigned to each yod in the 41×41 array. Then:

- the sum of these numbers assigned to the 168 blue yods surrounding the central yod = 4×168 = 672. This is the number of yods making up the first four Platonic solids when their faces and their internal triangles are Type A polygons (Fig. H). *The number 168 is the number of yods needed on average to construct from tetractyses the Platonic solids associated with the four physical Elements*;
- the sum of these numbers assigned to the 840 blue and green yods = 4×840 = 3360. This is also the sum of the number 4 assigned to the 840 red yods. Hence, the total sum = 3360 + 3360 = 6720. Compare this with the 4_{21} polytope, This 8-d, semiregular polytope is also known as one of the three so-called "Gosset polytopes," named after the amateur British mathematician Thorold Gosset, who first published his work on them in 1900. Its 240 vertices are now known to represent the 240 non-zero roots of E_8. It has 6720 edges, each half of the polytope having 3360 edges (Fig. I). The 41×41 array of the number 4 can be thought of as a representation of these edges, the 840 blue & green yods with the Tetrad assigned to them representing the 3360 edges in one half of the 4_{21} polytope and the 840 red yods representing its other half. In terms of the UPA, whose 10 whorls have 3360 turns in each revolution, every turn being a superposition of two perpendicular plane waves 90° out of phase, the two sets of 840 yods, when weighted with the Tetrad, correspond to the 3360 waves and their 3360 out-of-phase counterparts. The blue square with 168 yods weighted with the Tetrad generates the number 672, which is the number of plane waves making up each revolution of of a whorl. The numbers of blue, green & red yods are in the proportion 1:4:5. "1" corresponds to a single half-revolution of a whorl, "1+4" (= 5) refers to its outer or inner half, comprising 2½ revolutions (i.e., "4" refers to its two more revolutions), and "5" refers, respectively, to its inner or outer half, which comprises five half-revolutions.

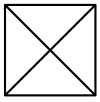
Assigning the Decad (10) to yods in the square arrays

Suppose that the number 10 symbolised by the tetractys is assigned to each yod in the 41×41 array. Then:

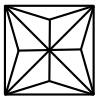
- the sum of the 1680 numbers associated with the 1680 red, green & blue yods = 16800. This is the number of turns in the 10 helical whorls of the UPA;
- the sum of the 168 numbers making up the blue square = 1680. This is the number of turns in each whorl of the UPA.
- the sum of the 672 numbers assigned to the 672 green yods = 6720. This is the number of edges of the 4_{21} polytope (Fig. I).
- there are nine times as many red & green yods as there are blue yods. Curiously, the 4_{21} polytope has 60480 triangular faces, which is *nine* times its number of edges (see here).

As one might expect, given the number 10 is the measure of a complete, holistic system, making it the weight for all 1680 yods in the square arrays generates not only the number of edges but also the number of faces of the 4_{21} polytope! This cannot be yet another highly improbable coincidence. It stretches credulity to breaking point to imagine that the same number (6720) could appear by chance in both the geometry of the 4_{21} polytope, which represents the 240 roots of the symmetry group E_8 appearing in $E_8 \times E_8'$ heterotic superstring theory, and the UPA, being the number of plane waves running through its 10 whorls during one revolution. Rather, it is, surely, evidence that the UPA *is* an $E_8 \times E_8'$ heterotic superstring? The first four Platonic solids provide further confirmation of this by their displaying the *same* sequence of numbers of geometrical elements that surround their axes: 168, 336, 336 & 840, i.e., 168, 672 (= 4×168) & 840 (= 5×168), totalling 1680 (= 10×168).

Figure J



Type A square



Type B square

The three prime numbers 13, 29 & 41 that measure the square arrays are themselves generated by the square. The Type A square has 13 points & lines and the Type B square has 29 points & lines and 41 points, lines & triangles (Fig. J).

As previously discussed and illustrated in Figure C, the 36 triangles making up the Type C square, when turned into tetractyses, contain 168 yods that surround its centre. This is the number of yods surrounding the central yod in the 13×13 array. Weighted with the Tetrad, the yods in the Type C square generate the number 672. This is the extra number of yods surrounding the centre of the 29×29 array. Weighted with the Decad (10), they generate the number 1680, which is the number of yods surrounding the centre of the 41×41 array.

The number 41 is the 21st odd integer, where 21 is the gematria number value of EHYEH (אהיה), the Godname assigned to Kether, the first Sephirah of the Kabbalistic Tree of Life. The number 29 is the 15th odd integer, where 15 is the number value of YAH (יה), the shortened, often poetic, form of the full Godname YHVH (יהוה) which is assigned to Chokmah, the second Sephirah. Is it just coincidence that the Godnames of *successive* Sephirah in the Tree of Life prescribe in an arithmetic way the two largest square arrays? No. True to their archetypal nature, *all* Godnames prescribe in various mathematical ways any parameter that quantifies a system conforming to the divine blueprint, whether it be the Kabbalistic Tree of Life or another sacred geometry. Examples of this universal power of prescription can be found in many sections of the author's website, as well as his research articles linked to on its homepage.

4×4 square array representations of 496

The dimension of $E_8 \times E_8'$ is 496. This is the gematria number value of Malkuth, the tenth Sephirah, signifying in a cosmic context the physical universe (see here). As 496 = 16×31, where 31 is the gematria number value of EL (אל), the Godname of Chesed, meaning "God", this number at the heart of superstring theory is the sum of a 4×4 array of the number 31:

$$496 = \begin{matrix} 31 & 31 & 31 & 31 \\ 31 & 31 & 31 & 31 \\ 31 & 31 & 31 & 31 \\ 31 & 31 & 31 & 31 \end{matrix}.$$

496 is the sum of the first 31 integers:

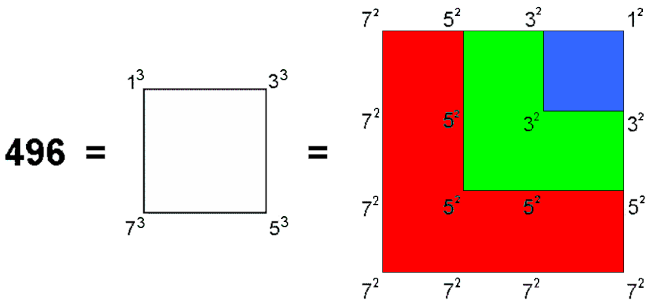
$$1 + 2 + 3 + \ldots + 31 = 496,$$

i.e., it is the 31st triangular number.‡

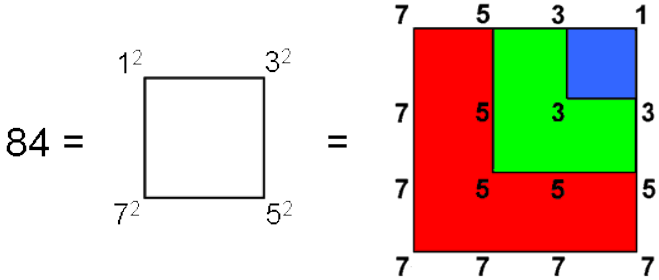
As

$$496 = 1^3 + 3^3 + 5^3 + 7^3,$$

This number can be represented by a 4×4 square array of the squares of the first four odd integers:



There are 84 circular turns in every quarter-revolution of a whorl. This number has an analogous representation:



Mathematicians call a number "highly composite" if it has more divisors than all integers smaller than itself. The superstring structural parameter 1680, which we have seen is both the population of the 41×41 array of yods and the number of geometrical elements surrounding the axes of the first four Platonic solids, was ascertained

‡ It is ironic that the mysterious number 496 so central to superstring theory should be determined by the ancient Hebrew word for God. Those who have doubted the veracity of superstring theory should take note.

by C.W. Leadbeater through painstaking counting of turns in the helical whorls of UPAs (he counted 135 different specimens!). It is the 16th member of this class of numbers, i.e., it is the largest number in a 4×4 array of the first 16 highly composite numbers:

2	4	6	12
24	36	48	80
120	160	240	380
720	840	1260	1680

Conclusion

Concentric 13×13, 29×29 and 41×41 square arrays of yods generate certain structural parameters of the UPA, a basic constituent of atoms that was allegedly remote-viewed by Annie Besant & C.W. Leadbeater over several decades, starting in 1895. This particle is interpreted by the author as the lightest subquark state of the $E_8 \times E_8'$ heterotic superstring. From a purely arithmetic point of view, this representation is not necessarily significant in itself because it could be just coincidence that there are several integers whose squares generate these parameters. What, however, makes these particular square arrays truly significant (and therefore of interest to theoretical physicists) is that exactly the *same* sequence of integers appears for the numbers of geometrical elements surrounding the axes of the first four Platonic solids when their faces and interiors are constructed from triangles. It is highly implausible that this, too, could arise by chance. The implication that some deep connection between the Platonic solids and superstrings, rather than simple coincidence, is the reason for their appearance is strengthened by the fact that the most basic structural parameter of the UPA, namely, the number 168, is both the number of geometrical elements surrounding the axis of the tetrahedron (the simplest Platonic solid) and the number of yods needed on average to construct from tetractyses one of the first four Platonic solids. These have always been regarded as possessing sacred geometry ever since the ancient Greeks thought that they were the shapes of the particles of the physical Elements of Fire, Air, Earth & Water. Moreover, in Kabbalah, the number 168 is the gematria number value of the Kabbalistic name of the Mundane Chakra of Malkuth (the tenth Sephirah), one of whose meanings is the physical universe, which some physicists believe is composed of superstrings. The square is an ancient symbol for the four Elements. It is therefore appropriate that it embodies not only this structural parameter of the UPA but also the dimension 248 of the exceptional Lie group E_8 whose symmetries define the unified force acting between superstrings of this type. These properties can be regarded as evidence for the author's identification of the UPA with this type of superstring. Further support for this is provided by two remarkable facts:

1. the average Platonic solid has 496 geometrical elements other than vertices surrounding its axis, where 496 is the gematria number value of Malkuth, so that the 248 geometrical elements in each half that are not vertices are analogous to the 248 roots of E_8 *and its copy* E_8';
2. the faces of the first four Platonic solids are composed of 248 points & lines when they are Type A polygons.

The number 4 (Tetrad) and the square that symbolises it express both arithmetically and geometrically the structural and dynamical parameters of the $E_8 \times E_8'$ heterotic superstring when the UPA is interpreted as one of the states of this particle. Their properties illustrate a powerful principle that the author has called the "Tetrad Principle." Discussed further in Article 1, It states that fundamental, mathematical parameters of the cosmos — both physical *and* superphysical — are always either:

- the *fourth* member of a class of numbers;
- embodied in the *fourth* member of a class of mathematical objects;
- expressed by the first *four* integers 1, 2, 3 & 4 that are symbolised by the four rows of dots in the Pythagorean tetractys;
- embodied in the square as the symbol of the Tetrad when its sectors are Type A triangles, Type B triangles, etc. or 1st-order tetractyses, 2nd-order tetractyses, etc. or when it is just an array of dots.