

The Holistic Nature of the 16-cell

by

Stephen M. Phillips

Flat 4, 117-119 West Hill Road. Bournemouth. Dorset BH2 5PH. England.

Website: www.smphillips.mysite.com

Abstract

The 16-cell is the 4-dimensional counterpart of the octahedron. This article presents evidence for its holistic character by demonstrating that all the gematria number values of the 10 Sephiroth, their Godnames, Archangels, Orders of Angels & Mundane Chakras quantify its geometrical and yod composition when it is constructed from Type A triangles. These 50 parameters emerge so naturally in the analysis as to make it highly improbable that their appearances are all due to chance. The holistic parameter 384 manifests in the 16-cell as its 384 rotations & rotations-reflections. Each half of the 16-cell consists of 84 triangles (48 in its faces, 36 in its interior). This 48:36 division of the holistic parameter 84 is characteristic of various sacred geometries, as well as quantifying Pythagorean intervals between the notes in the four Authentic Modes and the three distinct Plagal Modes. The 84:84 division of its triangles has a counterpart in the 84 intrinsic corners of the 141 triangles associated with each half of the inner Tree of Life when composed of Type B polygons, as well as in each half composed of Type A polygons as the 84 geometrical elements outside the root edge in either the triangle, square, pentagon & dodecagon or the hexagon, octagon & decagon that surround the five centres that are not corners of polygons. The 16-cell embodies the number 137 determining the fine-structure constant and the dimension 496 of the two symmetry groups $SO(32)$ & $E_8 \times E_8$ governing heterotic superstring forces that are free of quantum anomalies. The yod population of the 16-cell is the sum of the holistic parameters 384 and 248, where the latter is the dimension of E_8 . It indicates that the 16-cell embodies the root composition of E_8 and $E_8 \times E_8$. This confirms its holistic status because previous articles have proved that other sacred geometries embody these symmetry groups.

	SEPHIRAH	GODNAME	ARCHANGEL	ORDER OF ANGELS	MUNDANE CHAKRA
1	Kether (Crown) 620	EHYEH (I am) 21	Metatron (Angel of the Presence) 314	Chaioth ha Qadesh (Holy Living Creatures) 833	Rashith ha Gilgalim First Swirlings. (Primum Mobile) 636
2	Chokmah (Wisdom) 73	YAHWEH, YAH (The Lord) 26, 15	Raziel (Herald of the Deity) 248	Auphanim (Wheels) 187	Masloth (The Sphere of the Zodiac) 140
3	Binah (Understanding) 67	ELOHIM (God in multiplicity) 50	Tzaphkiel (Contemplation of God) 311	Aralim (Thrones) 282	Shabathai Rest. (Saturn) 317
	Daath (Knowledge) 474				
4	Chesed (Mercy) 72	EL (God) 31	Tzadkiel (Benevolence of God) 62	Chasmalim (Shining Ones) 428	Tzadekh Righteousness. (Jupiter) 194
5	Geburah (Severity) 216	ELOHA (The Almighty) 36	Samael (Severity of God) 131	Seraphim (Fiery Serpents) 630	Madim Vehement Strength. (Mars) 95
6	Tiphareth (Beauty) 1081	YAHWEH ELOHIM (God the Creator) 76	Michael (Like unto God) 101	Malachim (Kings) 140	Shemesh The Solar Light. (Sun) 640
7	Netzach (Victory) 148	YAHWEH SABAOTH (Lord of Hosts) 129	Haniel (Grace of God) 97	Tarshishim or Elohim 1260	Nogah Glittering Splendour. (Venus) 64
8	Hod (Glory) 15	ELOHIM SABAOTH (God of Hosts) 153	Raphael (Divine Physician) 311	Beni Elohim (Sons of God) 112	Kokab The Stellar Light. (Mercury) 48
9	Yesod (Foundation) 80	SHADDAI EL CHAI (Almighty Living God) 49, 363	Gabriel (Strong Man of God) 246	Cherubim (The Strong) 272	Levanah The Lunar Flame. (Moon) 87
10	Malkuth (Kingdom) 496	ADONAI MELEKH (The Lord and King) 65, 155	Sandalphon (Manifest Messiah) 280	Ashim (Souls of Fire) 351	Cholem Yesodoth The Breaker of the Foundations. The Elements. (Earth) 168

Table 1. Gematria number values of the ten Sephiroth in the four Worlds.
(All numbers in this table that the article refers to are written in **boldface**).

1. The 16-cell

Absent from the seven types of regular polygons making up the inner form of the Tree of Life are the heptagon, the nonagon and the undecagon. [Article 58](#) explored the analogy between these three absent polygons and the three Sephiroth of the Supernal Triad, as implied by the analogy between the seven polygons and the seven Sephiroth of Construction. [Article 59](#) revealed that the three absent polygons embody the root composition of E_8 and $E_8 \times E_8$. This remarkable property has been encountered in other sacred geometries and analysed in many earlier articles. It has revealed the 16-cell to be — *in that context* — the most fundamental of the six polychorons because the 24-cell that they also encode is a compound of three 16-cells. It is worthwhile to analyse its properties here in order to uncover evidence of its holistic character. This will manifest by the appearance of the gematria number values of the 10 Sephiroth in the four Kabbalistic Worlds quantifying its properties and by its displaying structural parameters of sacred geometries established elsewhere in this website. These numbers are listed in Table 1 shown above.

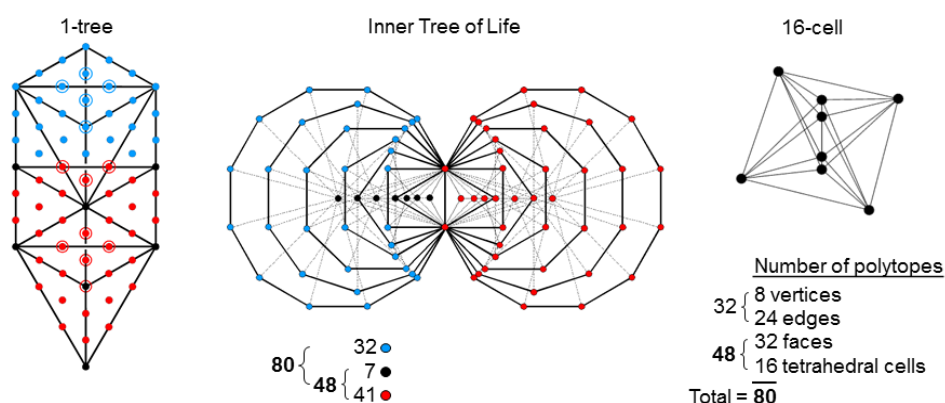
The 16-cell is the 4-dimensional version of the octahedron. It is composed of:

- eight points (vertices, or 0-polytopes) with coordinates ± 1 along each of the four perpendicular Cartesian axes; each is met by six edges, 12 faces & eight cells;
- 24 lines (edges, or 1-polytopes, joining all pairs of vertices except diametrically opposite ones; each one is met by four faces & four cells);
- 32 triangles (faces, or 2-polytopes); each is met by two cells;
- 16 tetrahedra (cells, or 3-polytopes); four meet at each edge.

It has **80** 0-, 1-, 2- & 3-polytopes, where **80** is the number value of Yesod ("Foundation"). This is the number of yods in the 1-tree constructed from tetractyses and the number of corners of the $(47+47=94)$ sectors of the $(7+7)$ enfolded polygons that are its inner form (see [here](#)). As evidence that the appearance of this number in the 16-cell is not coincidental but indicative of its holistic nature because it is described by the system of gematria number values of the 10 Sephiroth, observe that the distinction between the seven Sephiroth of Construction and the rest of the 1-tree generates the division:

$$80 = 32 + 48$$

in yods (**48** yods up to Chesed, the first Sephirah of Construction, and 32 blue yods above it). This appears in the inner Tree of Life as the **48** sector corners that are either black centres of the seven enfolded polygons in one set (see picture below) or the 41 red corners of the other set of seven enfolded polygons and as the 32 blue corners of the former set of seven polygons. It appears in the 16-cell as its 32 vertices & edges and its **48** faces & cells (Fig. 1):



Correspondence between the 1-tree, the inner Tree of Life & the 16-cell

Figure 1

It would be incorrect to regard the 32 faces of the 16-cell as corresponding to the 32 yods above Chesed because one would, intuitively, expect the degree of complexity of polytopes in the 16-cell to increase with the downward descent from Kether towards Malkuth of the yods that symbolise them, but this correspondence disrupts the order by placing 2-polytopes before 1-polytopes, instead of the more natural progression: 0-polytopes→1-polytopes→2-polytopes→3-polytopes, which has been followed in Figure 1.

Suppose that the 16-cell is constructed both externally and internally from Type A triangles. The geometrical & yod compositions of the six polychorons are discussed [here](#). The two compositions of the 16-cell are reproduced below in Table 2:

C/C'/c = number of corners; S/S'/s = number of sides; T/T'/t = number of triangles; N/N'/n = total number of geometrical elements;

H/H'/h = number of hexagonal yods; B/B'/b = number of boundary yods; Y/Y'/y = total number of yods.

Table 2.

Geometrical composition of the 16-cell

Surface				Interior				Total			
C	S	T	N	C'	S'	T'	N'	c	s	t	n
40	120	96	256	1+24	80	72	1+176	1+ 64=65	200	168	1+432

Yod composition of the 16-cell

Surface				Interior				Total			
C	H	B	Y	C'	H'	B'	Y'	c	h	b	y
40	336	280	376	1+24	232	1+184	1+256	1+ 64=65	568	1+464	1+632

("1" denotes the centre of the 16-cell).

Comments

1) Its 32 triangular faces contain 256 geometrical elements, where $256 = 4^4$. They comprise:

- 8 vertices, where $8 = 4^{\text{th}}$ even integer;
- 24 edges, where $24 = 1 \times 2 \times 3 \times 4 = 5^2 - 1 = 3 + 5 + 7 + 9 = \text{sum of 1st 4 odd integers after 1}$;
- **64** vertices, edges & faces, where $64 = 4^3$;
- 16 tetrahedral cells, where $16 = 4^2$;
- 40 corners (points), where $40 = (1+2+3+4) \times 4 = 4 + 8 + 12 + 16$;
- 120 sides (lines), where $120 = 2^2 + 4^2 + 6^2 + 8^2 = \text{sum of squares of 1st 4 even integers}$;
- 96 triangles, where $96 = 4 \times 24 = 4 \times 1 \times 2 \times 3 \times 4 = 4(5^2 - 1) = 4(3+5+7+9) = 12 + 20 + 28 + \mathbf{36}$;
- 160 corners & sides, where $160 = (1+2+3+4) \times 4^2$;
- 136 corners & triangles, where $136 = 16^{\text{th}}$ triangular number and $16 = 4^{\text{th}}$ square number;
- **216** sides & triangles, where $216 = 6^3$ and 6 is the *fourth* integer after 2, the first prime number.

This demonstrates very clearly how the Tetrad (4) expresses the geometrical composition of the faces of the 16-cell. It also expresses its interior, for there are 176 (=4×44) points, lines & triangles and 256 (=4⁴) yods inside the 16-cell surrounding its centre. There are **248** geometrical elements other than the eight vertices in its 32 faces, where **248** is the number value of *Raziel*, the Archangel of Chokmah. *The dimension 248 of the rank-8 exceptional Lie group E₈ is embodied in the facial geometry of a single 16-cell.* The **248** geometrical elements comprise 128 corners & triangles and 120 sides. Its faces have (120+96=**216**) triangles & their sides, where **216** is the number value of Geburah. As 432 geometrical elements surround its centre, each half of the 16-cell contains **216** geometrical elements. The 16-cell has **168** triangles with **65** corners, where **65** is the number of ADONAI, the Godname of Malkuth, and **168** is the number of *Cholem Yesodoth*, the Mundane Chakra of *this* Sephirah. This conjunction of two gematria numbers associated with the *same* Sephirah is both remarkable and highly significant. 432 geometrical elements surround its centre ("1"), demonstrating once again the role of the integers 1, 2, 3 & 4 symbolised by the tetractys in expressing properties of the 16-cell. 264 corners & sides (**64** corners, 200 sides) surround its centre, where 264 is the number of yods in the seven enfolded Type A polygons that make up the inner Tree of Life (see [Article 64](#), Table 5, and [here](#)). This embodiment of a primary parameter of the 7 enfolded Type A polygons of the inner Tree of Life is *unmistakable evidence* of the Tree of Life/holistic character of the 16-cell. It is confirmed by the fact that 24 of the 200 sides are edges of the faces of the 16-cell, leaving 176 sides of the **168** triangles that are not edges. Compare this with the fact that the seven enfolded polygons have 176 corners, sides & triangles (see [here](#)). In both cases, the holistic structural parameter 264 divides up in the *same* way:

7 enfolded polygons: 264 yods = 176 hexagonal yods on 88 sides + 88 corners & centres of 47 tetractyses;

16-cell: 264 corners & sides = 176 sides + (**64**+24=88) corners & edges.

The four yods in the root edge of the seven enfolded polygons correspond to any two opposite vertices and the two sides of internal triangles that form an axis of the 16-cell (see (6) below). The 260 (=26×10)

yods outside the root edge correspond to the 260 corners & sides outside this axis: $260 = 62$ corners + 198 sides = 6 vertices + 56 corners + 24 edges + 174 sides. This is how YAHWEH with number value **26** prescribes the 16-cell. Just as the 264 yods in the 7 enfolded polygons show the factorisation: $264 = 3 \times 88$ as 2×88 (hexagonal yods) + 88 (corners & centres of tetractyses), so, too, the 264 corners & sides of the 16-cell comprise the 88 sides in each half that are not edges and the 88 corners & edges of the whole polychoron. *Isomorphism exists between the inner Tree of Life and the 16-cell because the latter is the 4-dimensional manifestation of the former.*

2) The 16-cell has **64** vertices, edges & faces, where **64** is the number value of *Nogah*, the Mundane Chakra of Netzach. It also has **64** corners of **168** triangles surrounding its centre. When its faces and interior triangles are simple triangles rather than Type A triangles, there are in its interior 24 internal triangles with one corner and eight sides, totalling nine corners, 32 sides & 56 triangles, i.e., **97** internal geometrical elements, where

$$97 = 21 + 26 + 50$$

is the sum of the Godname numbers of Kether, Chokmah & Binah; it is also the number value of *Haniel*, the Archangel of Netzach. When these triangles are tetractyses, there are $(1 + 8 \times 2 + 24 = 41)$ internal yods and $(8 + 24 \times 2 + 32 = 88)$ yods in its faces, a total of **129** yods, where **129** is the number value of YAHWEH SABAOTH, the Godname of Netzach. Nine yods are corners of 56 tetractyses, leaving 120 hexagonal yods, where

$$120 = 11^2 - 1 = 3 + 5 + 7 + \dots + 21$$

is the sum of the first *ten* odd integers after 1, showing how the Decad (and EHYEH with number value **21**) determines the number of hexagonal yods needed to construct the 16-cell from tetractyses. 336 of the 632 yods surrounding the centre of the 16-cell with Type A triangles are hexagonal yods in its faces, leaving 296 yods (**148** in each half) that are either corners or internal hexagonal yods. **148** is the number value of Netzach. Here is a conjunction of four numbers associated with the *same* Sephirah (Netzach). $(96+16=112)$ points, lines, triangles & tetrahedra surround its centre, where **112** is the number value of *Beni Elohim*, the Order of Angels assigned to Hod. The 16-cell has **48** faces (2-polytopes) & tetrahedral cells (3-polytopes), where **48** is the number value of *Kokab*, the Mundane Chakra of Hod.

3) **280** yods line the 96 tetractyses in its 32 faces. This is the number value of *Sandalphon*, the Archangel of Malkuth. Here is a second, remarkable, chance-defying conjunction in the *same* context of the gematria number values of the Godname (**65**), Archangel (**280**) & Mundane Chakra (**168**) of the *same* Sephirah (Malkuth). **140** yods line the **48** tetractyses making up the 16 faces in each half of the 16-cell, where **140** is the number value of *Masloth*, the Mundane Chakra of Chokmah. 376 yods make up its faces (188 in each half). **187** yods in the 16 faces in each half surround its axis, where **187** is the number value of *Auphanim*, the Order of Angels assigned to Chokmah. As (1) shows that the 16-cell embodies the number **248** of *Raziel* and the number **26** of YAHWEH and as (4) shows that it embodies the number **73** of Chokmah, here is a conjunction of all five numbers associated with the *same* Sephirah.

4) When its faces are simple triangles, they are lined by $(8 + 24 \times 2 = 56)$ yods; $(1 + 8 \times 2 = 17)$ yods line the 24 internal triangles formed by joining vertices to the centre of the 16-cell. $(56+17=73)$ yods line its $(24+32=56)$ triangles, **72** such yods surrounding its centre. The 16-cell embodies the number value **73** of Chokmah as the number of yods that line all its tetractyses, both externally and internally. **73** is the **21st** prime number, showing how EHYEH, the Godname of Kether with number value **21**, prescribes in a minimal way the *form* of the 16-cell. The number of yods in the 32 faces of the 16-cell = $56 + 32 = 88$. There are **80** hexagonal yods, where **80** = **48** (lying on 24 edges) + 32 (at centres of 32 faces). It should be noticed that this division is the characteristic division of the **80** yods of the 1-tree and the **80** corners of the 94 sectors in its polygonal inner form that was discussed above. The double presence of this number in both the geometrical and the yod composition of the 16-cell is evidence that it acts as the polytopical "foundation." This another reason why it should be regarded as fundamental, i.e., the "hydrogen atom" of the polychorons, even though it is not the polychoron with the least number of vertices.

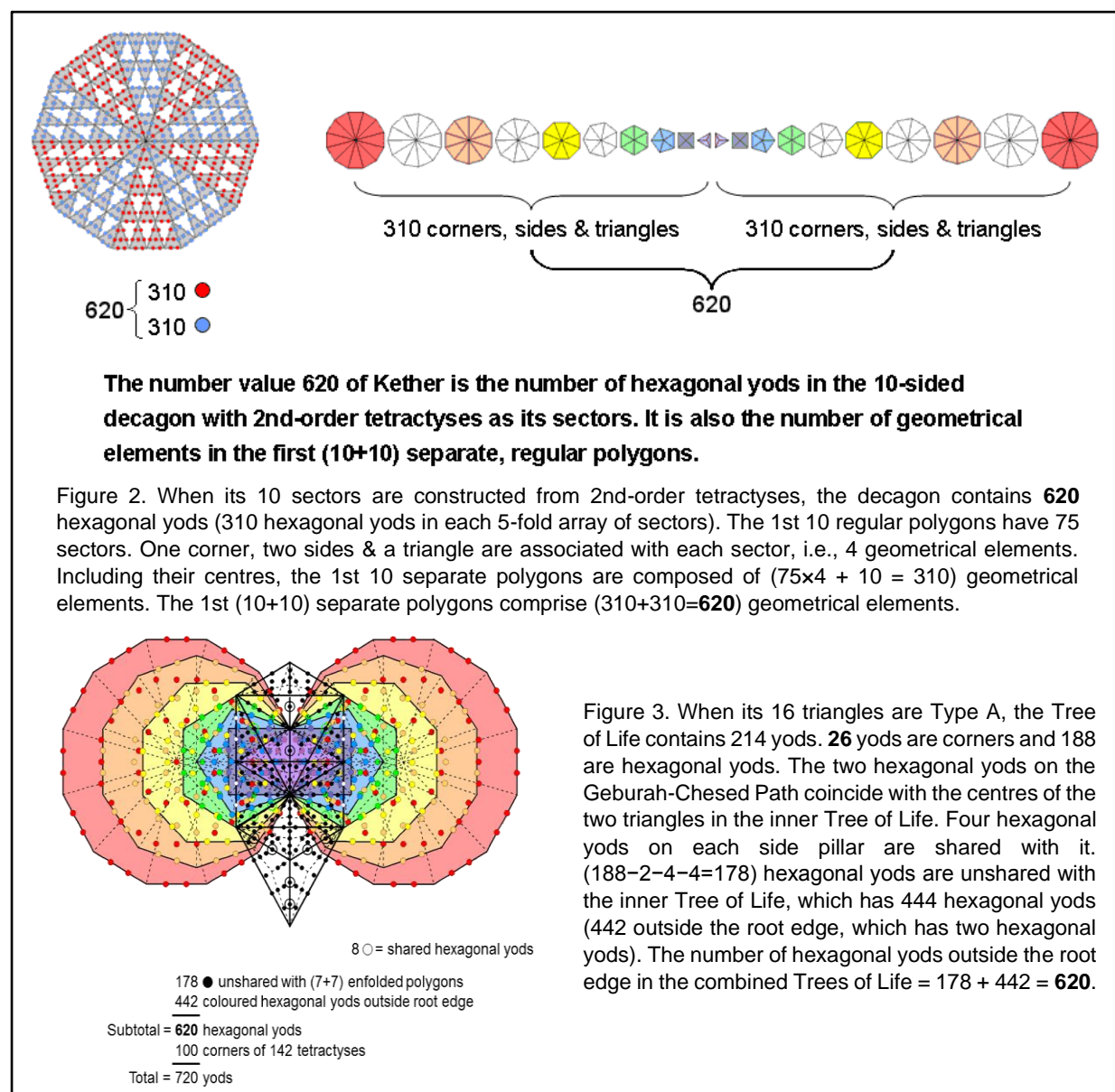
5) When its faces and interior triangles are simple triangles, there are $(8+24=32)$ corners & sides of triangles in its faces and $(1+8=9)$ internal corners & sides, a total of 41 corners & sides. This is both the **21st** odd integer and the **15th** prime number, showing how EHYEH, the Godname of Kether with number value **21** and YAH, the shortened Godname of Chokmah with number value **15**, prescribe the minimal geometry of the 16-cell. Collecting together results discussed in (1), (3) & (4), all six numbers associated with the Sephirah Chokmah are now seen to be naturally embodied in the 16-cell.

6) If we consider an axis passing through two opposite vertices and its centre, it is composed of three corners and two sides of triangles. Therefore, $(41-3-2=36)$ corners & sides surround it. This is the number value of *ELOHA*, the Godname of Geburah. 632 yods surround the centre of the 16-cell. This means that,

given its centre and two "poles" (a pair of vertices at ± 1 on the axis), **630** yods are needed to construct its **168** tetractyses. **630** is the number value of *Seraphim*, the Order of Angels assigned to Geburah. The **97** points, lines & triangles include two lines forming this axis. The 16-cell contains **95** points, lines & triangles other than these two axial lines, where **95** is the number value of *Madim*, the Mundane Chakra of Geburah. Surrounding the centre of the 16-cell with Type A triangles are 264 corners & sides, i.e., 262 geometrical elements other than these two lines. Each half has **131** such geometrical elements. This is the number value of *Samael*, the Archangel of Geburah. The 16-cell embodies all five number values associated with Geburah.

7) The 32 faces of the 16-cell contain 336 hexagonal yods (96 at centres of tetractyses, 240 lining their 120 sides). This number is discussed widely on this website as a superstring structural parameter embodied in sacred geometries, being the number of turns in one revolution of each helical whorl of a UPA (subquark state of the $E_8 \times E_8$ heterotic superstring) around its axis of spin. Together with the above examples of how the Godnames prescribe its properties, here is irrefutable evidence that the 16-cell is a holistic object, for it is highly improbable that a number appearing in so many sacred geometries would turn up in this context just by chance. Rather, the number 336 does so in the 16-cell because it, too, is holistic, so that this number *must* quantify at least one of its properties, viz., how many hexagonal yods are needed to construct its faces out of Type A triangles.

8) The symmetry group of the **n-cube** and its dual, the n-dimensional **cross-polytope**, is B_n , and is known as the **hyperoctahedral group**. It has order $2^n n!$. For $n = 4$, this is 384. B_4 has the subgroup D_4 with order 192. This is the group of rotations of the 4-cube and the 16-cell. D_4 has the subgroup D_3 , which is the



group of rotations of the cube and octahedron. The holistic parameters 384 and 192, which show the divisions:

$$384 = 48 + 336$$

and

$$192 = 24 + 168,$$

signify in the present context the 384 rotations+reflections of the 4-cube and 16-cell, the **48** rotations+reflections of their 3-dimensional counterparts — the cube and the octahedron, the 192 proper rotations¹ of the 4-cube and 16-cell and the 24 proper rotations of the cube and octahedron. The appearance of the holistic parameters 384 & 192 as orders of the hyperoctahedral group for 4-polytopes does not mean that all sacred geometries that embody these numbers depict only the symmetries of 4-dimensional objects. For example, the five Platonic solids embody them (see [here](#)), as do the seven separate polygons making up the inner form of the Tree of Life (see [here](#)). What it *does* mean is that such objects, in which these numbers uniquely appear, are holistic in character, exhibiting the *complete* pattern of the Whole, even though they may be only components of it.

9) 632 yods surround the centre of the 16-cell constructed from Type A triangles. There are four hexagonal yods lining the axis. Surrounding the axis are (632-4-8=620) yods that are not vertices. In other words, **620** yods need to be added to create its shape from Type A triangles. This is the number value of Kether ("Crown"). It is:

1. the number of hexagonal yods in a decagon with 2nd-order tetractyses as sectors;
2. the number of geometrical elements in the first (10+10) polygons;
3. the number of hexagonal yods in the combined outer & inner Trees of Life constructed from Type A polygons.

(Figs. 2 & 3).

These examples demonstrate how the number **620** of Kether (the first Sephirah) is a parameter of holistic systems. 310 (=31×10) such yods in each half of the 16-cell surround this axis. **31** is the number value of EL ("God"), the Godname of Chesed. **62** corners of tetractyses surround the axis. This is the number value of *Tzadkiel*, the Archangel of Chesed. We found in (4) that **72** yods line the 56 tetractyses in either its faces or interior when its faces and internal triangles are tetractyses. **72** is the number value of Chesed. 432 geometrical elements surround its centre. The axis is composed of two vertices, two sides and the centre. Surrounding it are (432-2-2=428) geometrical elements, where **428** is the number value of *Chasmalim*, the Order of Angels assigned to Chesed. Here is another remarkable conjunction of gematria number values (this time, four) associated with the *same* Sephirah.

10) The number of yods in the 16-cell other than the five internal yods on its axis = 633 - 5 = 628. Each half of the 16-cell has **314** yods other than yods inside it on its axis. The number **314** is the number value of *Metatron*, the Archangel of Kether. The number of yods in the 16-cell other than vertices = 633 - 8 =

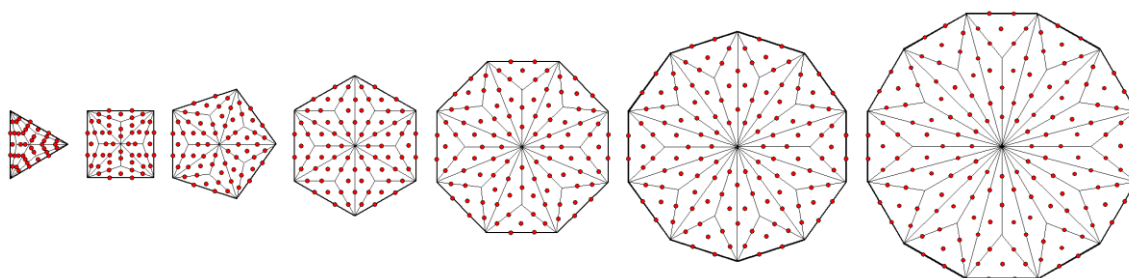


Figure 4. A Type B N-gon has (15N+1) yods that comprise (2N+1) corners & 13N hexagonal yods. The number of hexagonal yods in the 7 Type B polygons with 48 corners = $\sum 13N = 13 \times 48 = 624$. This is the number of yods needed to construct the 16-cell from Type A triangles, starting with its 8 vertices.

¹ Proper rotations are pure rotations about an axis unaccompanied by reflections. Improper rotations are rotations about an axis followed by reflection in a mirror perpendicular to that axis. For more details, see [here](#).

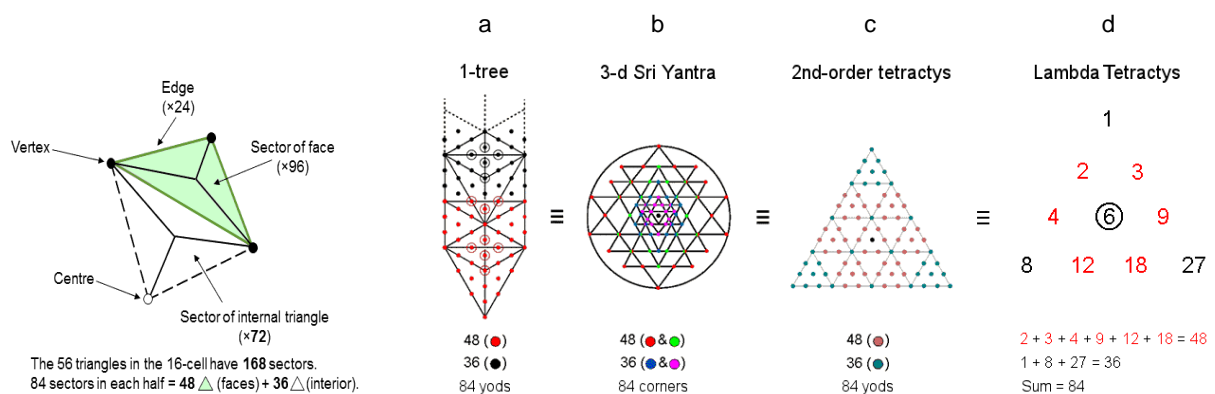
$625 = 5^4$. There are 624 such yods surrounding its centre. This is the number of hexagonal yods in the seven separate Type B polygons that make up the inner form of the Tree of Life (Fig. 4).

11) The 16-cell has 568 hexagonal yods. Four of them line the axis, so that $(568-4=564)$ hexagonal yods surround it. **282** such yods in each half of the 16-cell surround its axis. The number value of *Aralim*, the Order of Angels assigned to Binah, is **282**. Surrounding its centre are 464 yods lining sides of **168** tetractyses. Six yods apart from the centre lie on the axis. $(464-6=458)$ boundary yods surround the axis, 229 such yods in each half. 229 is the **50th** prime number, showing how ELOHIM, the Godname of Binah with number value **50**, prescribes the number of yods *shaping* the faces and interior of the 16-cell constructed from Type A triangles. 632 yods surround its centre, 316 yods being in each half. Including the centre, which is shared by both halves, there are **317** yods in each half. **317** is the number value of *Shabathai*, the Mundane Chakra of Binah. Excluding the two hexagonal yods on each half of the axis and the four vertices in each half of the 16-cell, there are $(317-2-4=311)$ yods in each half sharing the centre. **311** is the number value of *Tzaphkiel*, the Archangel of Binah. The axis has one corner and two sides inside the 16-cell. When they are simple triangles, the 32 faces comprise 32 corners & sides. The faces & axis comprise nine corners, **26** sides & 32 triangles, i.e., **67** geometrical elements, where **67** is the number value of Binah. The 16-cell embodies all five number values associated with this Sephirah.

The fact just proven above that all the Kabbalistic gematria number values of the 10 Sephiroth emerge naturally from analysis of the geometrical and yod compositions of the 16-cell is powerful evidence of the holistic nature of this polychoron. Had the appearance of these numbers been manufactured artificially by adding numbers that referred to unrelated contexts, the five numbers associated with the *same* Sephirah would be scattered about randomly in unconnected contexts. Instead, they often appear in the *same*, mathematical context with a frequency that exceeds chance. This is indicative of a *coherent design* or *conceptual order* that exists because the 16-cell is a holistic object. The gematria numbers quantify 10 sets of properties of the 16-cell, each set expressing the mathematical aspect of a Sephirah functioning in the four Kabbalistic Worlds of Atziluth, Beriah, Yetzirah & Assiyah. As we shall discover below, the 16-cell displays the *same* universal patterns in its structural parameters as those found in sacred geometries.

2. 36:48 divisions in the 16-cell and sacred geometries

Each half of the 16-cell has $(28 \times 3 = 84)$ sectors of 28 Type A triangles. The 16 faces in each half have



Equivalence of the 1-tree, the 3-d Sri Yantra, the 2nd-order tetractys and Plato's Lambda tetractys

- The 84 yods up to the top of the 1-tree consist of **48** red yods up to Chesed (the 1st Sephirah of Construction) and **36** black yods above it up to Kether.
- The 3-dimensional Sri Yantra has 42 triangles surrounding its central one with 84 corners. The 1st & 2nd layers with 18 triangles have **36** corners; the 3rd & 4th layers with 24 triangles have **48** corners.
- 84 yods in the 2nd-order tetractys surround its black centre. **36** green yods are either corners of tetractyses or in the 3 corner tetractyses. **48** brown hexagonal yods in the 7 tetractyses surround the black centre.
- The sum of the 9 integers in the Lambda Tetractys that surround its central integer 6 is 84. The sum of the black integers at the 3 corners is **36**. The sum of the six red integers is **48**.

Figure 5. The 16-cell is holistic because the **36:48** division of its triangles in each half exists in sacred geometries.

($3 \times 16 = 48$) sectors and its 12 internal Type A triangles have ($12 \times 3 = 36$) sectors. This **36:48** division in the 84 simple triangles making up each half of the 16-cell is characteristic of holistic systems that embody the number 84 as one of their defining parameters. Figure 5 displays examples.

7 Diatonic scales as the musical counterpart of the 16-cell

The *same* **36:48** division of the holistic parameter 84 appears in the 84 repetitions of Pythagorean intervals² between the notes of the seven diatonic musical scales making up the eight Church Modes. The four Authentic Modes have **48** repetitions of Pythagorean intervals between their notes, whilst the three

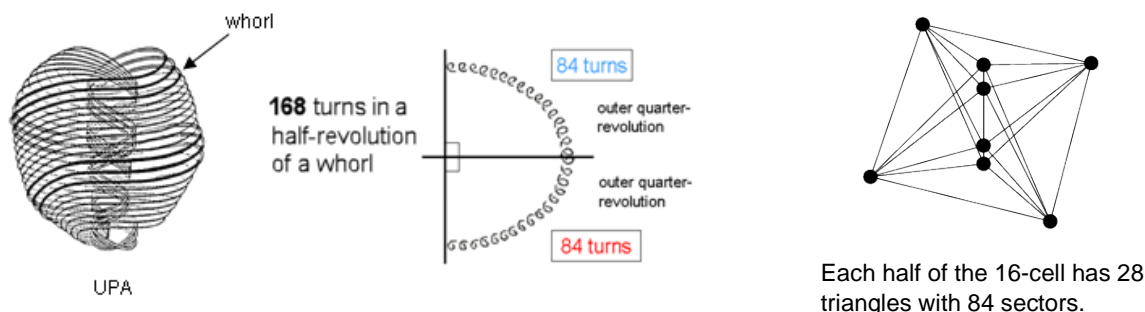


Figure 6. The 16-cell embodies the same structural parameter **168** as the UPA (subquark state of the $E_8 \times E_8$ heterotic superstring) because both are holistic systems that must be parameterised by the gematria number values of the 10 Sephiroth in the four Kabbalistic Worlds (in this case, the number value **168** of Cholem Yesodoth, the Mundane Chakra of Malkuth).

Plagal Modes have **36** repetitions of Pythagorean intervals between their notes (for proof, see the fourth comment under the table of intervals [here](#)). One half of the 16-cell with 84 triangles is analogous to the 84 repetitions of *rising*, Pythagorean intervals; its opposite half with 84 triangles corresponds to the 84 repetitions of *falling*, Pythagorean intervals (N.B. if a rising interval has a tone ratio of N ($N > 1$), its falling counterpart has a tone ratio of $1/N$). The **48** triangles making up the 16 faces in each half of the 16-cell are analogous to the **48** repetitions of Pythagorean intervals between the notes of the 4 Authentic Modes, whilst the **36** triangles inside each half of the 16-cell correspond to the **36** repetitions of Pythagorean intervals (rising or falling) between the notes of the three Plagal Modes. The distinction between the exterior and interior of the 16-cell is what corresponds to the difference between the four Authentic Modes and the three Plagal Modes in the seven diatonic scales.

The 84:84 division in the UPA/superstring

The 84 sectors in each half of the 16-cell has their counterpart in the UPA/superstring (Fig. 6). Each of its

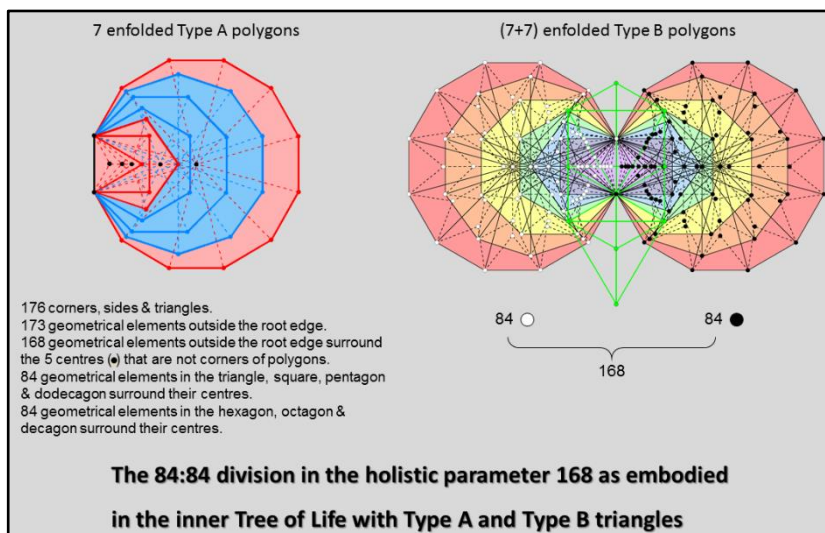


Figure 7. The 84:84 division in a half-revolution of a whorl of the UPA has its counterpart in the 7 enfolded Type A polygons as the 84 geometrical elements outside the root edge in either the triangle, square, pentagon & dodecagon or the hexagon, octagon & decagon that surround the 5 centres that are not corners of polygons. It appears in the (7+7) enfolded Type B polygons as the 84 corners of the 141 triangles associated with each set of 7 polygons.

² "Pythagorean intervals" are intervals that are also tone ratios of the 6 notes between the tonic and octave of the Pythagorean scale.

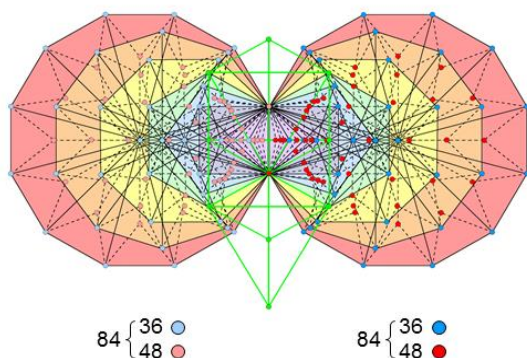
10 whorls is a helix with 1680 circular turns. The helix winds five times around its axis of spin, so that every half-revolution comprises **168** turns (84 per quarter-revolution).

The 84:84 division in the inner Tree of Life

As evidence that the 16-cell is a holistic object containing features analogous to those found in certain sacred geometries, consider the inner Tree of Life when its polygons are either Type A or Type B (Fig. 7).

Type A polygons

The seven enfolded Type A polygons have 176 corners, sides & triangular sectors (see Table 3_ [here](#)). Their shared root edge is a side with a corner at each end, i.e., it is composed of three geometrical elements, so that $(176-3=173)$ geometrical elements are outside the root edge. The centres of the hexagon and decagon are also corners of, respectively, the triangle and the pentagon. Therefore, five centres (shown as black dots in the picture above) are *not* corners of polygons. They are surrounded by



The 36:48 division in the 84 intrinsic corners of triangles associated with each set of 7 enfolded Type B polygons

Figure 8

$(173-5=168)$ geometrical elements outside the root edge. The centre of a Type A n -gon divided into its n sectors is surrounded by $4n$ geometrical elements. The separate Type A triangle, square, pentagon & dodecagon with 24 corners have $(4 \times 24 = 96)$ geometrical elements surround their centres. Enfolded, they have $(96 - 3 \times 3 = 87)$ such geometrical elements, i.e., $(87-3=84)$ geometrical elements surrounding centres of polygons are outside the root edge, where **87** is the number value of *Levanah*, the Mundane Chakra of Yesod. The hexagon, octagon & decagon with 24 corners have $(168-84=84)$ geometrical elements outside the root edge that surround their centres.

Type B polygons

The seven enfolded Type B polygons have 141 triangles with 88 corners. The $(7+7)$ enfolded Type B polygons making up the inner Tree of Life have **282** triangles with 174 corners, where **282** is the number value of *Aralim*, the Order of Angels assigned to Binah. Three corners lying on the vertical axis of the hexagon coincide with corners of triangles belonging to the outer form of the Tree of Life (coloured green in Fig. 8). $(174 - 2 \times 3 = 168)$ corners are *intrinsic* to the inner Tree of Life. Each endpoint of the root edge can be associated with one set of seven enfolded polygons. This means that 84 intrinsic corners can be associated with each set. They are the counterpart of the $(84+84)$ triangles in the two halves of the 16-cell. As the set of seven enfolded Type A polygons have 47 sectors with 41 corners, $(41-2-3=36)$ corners outside the root edge are intrinsic to each set. They are coloured dark blue or light blue in Fig. 8.

The remaining **48** corners consist of the corners at the centres of the 47 sectors and one of the two endpoints of the root edge; they are coloured either dark red or light red. The **36** light or dark blue corners correspond to the **36** triangles making up the 12 Type A triangles inside each half of the 16-cell, whilst the **48** dark red or light red corners correspond to the **48** triangles making up the 16 faces in each half when they are Type A triangles. We find that the 84 triangles making up each half of the 16-cell are the counterpart of these 84:84 divisions exhibited by the two halves of the inner Tree of Life. Furthermore, each set of 84 triangles displays the **36:48** division that is characteristic of this holistic parameter, as illustrated above for various sacred geometries, including the inner Tree of Life. *It is evidence that the 16-cell is the polychoron version of the archetypal structure of a holistic system.*

3. 36:48 & 84:84 divisions in the square representation of 624 & 168

As

$$13^2 - 1 = 168 = 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25,$$

the holistic parameter **168** is the sum of the first 12 odd integers after 1. As

$$25^2 - 1 = 624 = 3 + 5 + 7 + \dots + 49 = 168 + 27 + 29 + \dots + 49 = 168 + 456,$$

and a Type A square has 25 yods, the number of yods other than vertices (624) surrounding the centre of

a 16-cell constructed from Type A triangles is the sum of the 24 odd integers 3-49 that can be assigned to the 24 yods surrounding the centre of a Type A square (Fig. 9).

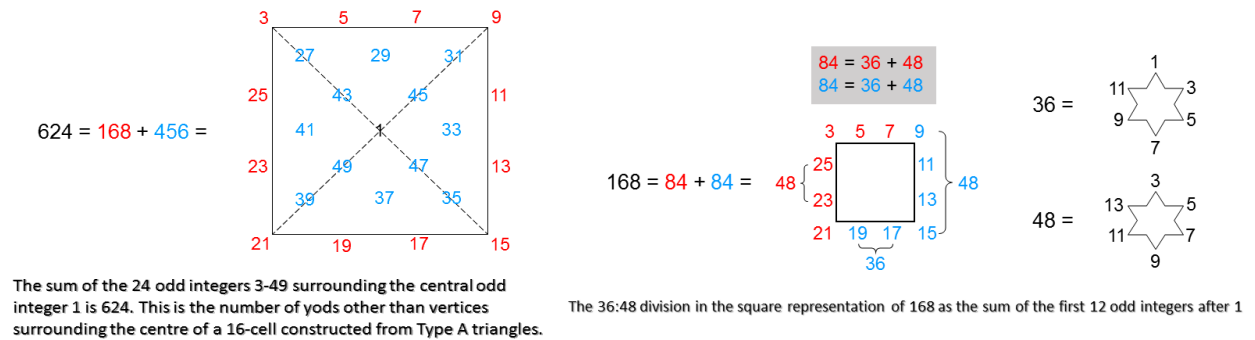


Figure 9

The sum of the first 12 odd integers after 1 assigned to the 12 yods on the boundary of the square is **168**, which is the number of hexagonal yods at the centres of its **168** tetractyses. The sum of the remaining 12 odd integers assigned to the 12 internal yods surrounding the centre of the square is 456, which is the number of yods other than vertices that line the sides of the **168** tetractyses. The central integer 1 denotes the centre of the 16-cell. Here is a remarkable arithmetic representation of the number of yods needed to construct the 16-cell from Type A triangles. What is also remarkable is that the red and blue, L-shaped sides of the square array each add up to 84, the integers in either one *naturally* adding up to **36** and **48** to generate arithmetically the **36:48** division that is characteristic of this holistic parameter. Moreover, **36** is the sum of the first 6 odd integers, whilst **48** is the sum of the first 6 odd integers after 1. The number of yods needed to construct the 16-cell from Type A triangles is not only the number of hexagonal yods in the 7 separate Type B polygons making up the inner Tree of Life but also the sum of the 24 odd integers after 1 that can be assigned to the 24 yods surrounding the centre of a Type A square. Instead of these congruencies between number and sacred geometry being dismissed as just a coincidence, it is more plausible to see them as yet more evidence for the holistic nature of the 16-cell.

4. 16-cell embodies the holistic parameters 137, 248, 384 & 496

A remarkable property of the yod population (633) of the 16-cell is:

$$137 + 496 = 633.$$

Many physicists regard the number 137 as one of the most mysterious numbers because its reciprocal measures approximately what they call the "**fine-structure constant**," which, in electrostatic c.g.s. units, is defined as $\alpha = e^2/\hbar c \cong 1/137$. Many sceptics of superstring theory view the dimension **496** of $E_8 \times E_8$ and $SO(32)$ (the only two gauge symmetry groups that leave superstring interactions free of **quantum anomalies**) as equally mysterious because no one has ever convincingly proposed a theory that derives either number from fundamental principles. The latter emerges only from imposing consistency of these interactions with quantum mechanics. Both numbers are parameters of sacred geometries (e.g., the yod population of the inner Tree of Life with Type B polygons is 1370, which is the number of yods in 137 tetractyses; see [here](#)). There is, therefore, no longer any mystery about either number because both of them characterise all holistic systems, including superstrings. For examples of their presence in sacred geometries, see [The holistic pattern](#) under the headings "137" and "496 = 248 + 248."

Another remarkable property of the number 633 is that:

$$385 + 248 = 633,$$

where

$$385 = 1^2 + 2^2 + 3^2 + \dots + 10^2$$

is the 10th square pyramidal number and **248** is the dimension of E_8 . The first 10 Type A polygons have 75 sectors lined by 385 yods (see Table 2 in [Article 58](#)). As

$$496 = 248 + 248,$$

$$\therefore 137 + 248 = 385.$$

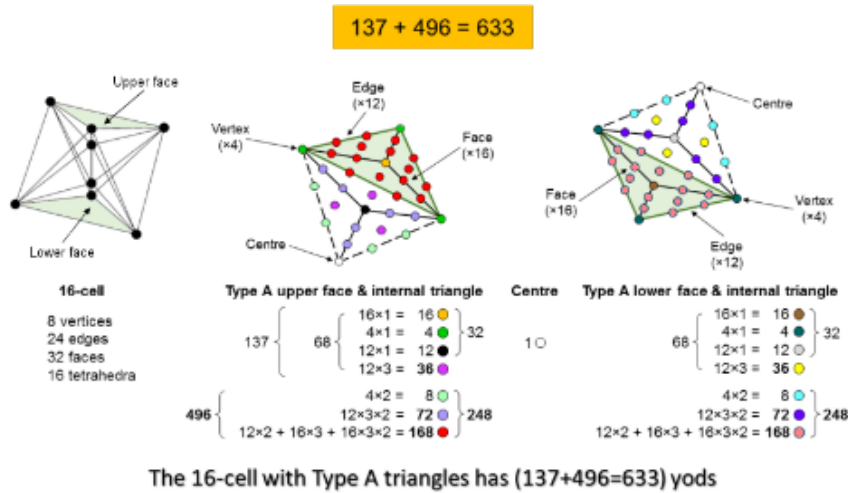


Figure 10

The number 385 is another holistic parameter, although the number 384 is, usually, the more visible signature of its presence, the extra "1" denoting merely the centre of the system. As 632 yods surround the 16-cell,

$$384 + 248 = 632.$$

The number of yods surrounding the centre of the 16-cell is the sum of the holistic parameters 384 & **248**.

Let us investigate what the first highlighted equation means for a 16-cell constructed from tetractyses. Figure 10 depicts a Type A triangular face of an upper and a lower half of the 16-cell, together with a Type A internal triangle with an edge as one side and the centre of the polychoron as its interior corner.

137

The upper half has 16 faces with four green vertices. Its 16 triangular faces have orange centres. Its 12 internal Type A triangles have black centres, three violet hexagonal yods at the centres of the sectors of each one and six dark grey hexagonal yods lining their sides. The lower half has 16 faces with four dark green vertices and a brown yod at the centre of each face. Its internal Type A triangles have light grey centres, three yellow hexagonal yods at the centres of their sectors and six mauve hexagonal lining their sides. Each half has $(3 \times 12 + 3 \times 16 = 84)$ tetractyses with 32 corners and **36** internal hexagonal yods (violet/yellow). Together with the centre of the 16-cell, there are $[1 + 2(32+36) = 137]$ yods that are either corners (**65**) or hexagonal yods (**72**) at the centres of internal tetractyses.

496

Four vertices in each half of the 16-cell are joined to the centre of the 16-cell by 4 sides with eight hexagonal yods (light green/turquoise). **72** hexagonal yods (dark grey/mauve) line the three tetractyses in each internal Type A triangle and **168** hexagonal yods (red or dark green) either line the tetractyses in the faces or are at their centres. Each half comprises $(8+72+168=248)$ yods that are not either corners or

The 240 non-zero roots of the superstring gauge symmetry group E_8

The roots of the E_8 Lie algebra are described in terms of eight orthonormal unit vectors $\{u_i\}$

Eight zero roots correspond to points at the centre of the root diagram and 240 non-zero roots all have length $\sqrt{2}$. They are given by

$$\pm u_i \pm u_j \quad (i, j = 1, 2, \dots, 8)$$

and

$$\frac{1}{2}(\pm u_1 \pm u_2 \pm \dots \pm u_8) \quad (\text{even number of '+'s})$$

Their explicit forms as 8-tuples and their numbers are listed below:

$$\begin{aligned} &(1, 1, 0, 0, 0, 0, 0, 0) \text{ and all permutations. Number} = \binom{8}{2} = 28; \\ &(1, -1, 0, 0, 0, 0, 0, 0) \text{ and all permutations. Number} = \binom{8}{2} = 28; \\ &(1, 1, 0, 0, 0, 0, 0, 0) \text{ and all permutations. Number} = 2 \times \binom{8}{2} = 56; \\ &(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \text{ and all permutations. Number} = \binom{8}{2} = 28; \\ &(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \text{ and all permutations. Number} = \binom{8}{2} = 28; \\ &(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \text{ and all permutations. Number} = \binom{8}{4} = 70; \\ &(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}). \text{ Number} = 1; \\ &(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}). \text{ Number} = 1. \end{aligned}$$

240

Figure 11

- **248** hexagonal yods \rightarrow **248** roots of E_8 ;
- 8 hexagonal yods on internal sides joining 4 vertices to the centre of the 16-cell \rightarrow 8 zero (simple) roots of E_8 ;
- **72** hexagonal yods on **36** sides of **36** internal tetractyses \rightarrow **72** roots of E_8 ;
- **168** hexagonal yods lining 60 sides of **48** tetractyses or at their centres in the 16 faces \rightarrow **168** roots of E_8 that are not roots of E_6 .
- 240 hexagonal yods in 16 faces or lining tetractyses inside internal Type A triangles \rightarrow 240 non-zero roots of E_8 .

centres of internal tetractyses. Both halves contain $(248+248=496)$ such yods. They correspond to the

(**248+248=496**) roots of $E_8 \times E_8$. Figure 11 shows correspondences between the **248** yods and the **248** roots of E_8 (the properties refer to each half of the 16-cell).

The hexagonal yod composition of the 16-cell is isomorphic to the **8:72:168** root composition of each E_8 in $E_8 \times E_8$, each half of the 16-cell containing **248** yods that correspond to the **248** roots in E_8 . This is amazing but not unexpected, for the 16-cell is, after all, a *holistic* object, so that it must display *all* the parameters that characterise such objects, one of which is the number **496**, whose origin is still so mysterious to superstring physics. Here we find it embodied in the 16-cell, along with the equally enigmatic number 137, whose reciprocal physicists have long known is approximately equal to the fine-structure constant, as discussed earlier. Their mystery vanishes once it is understood that they belong to a class of numbers that parameterise those geometrical objects (or, more generally, *holistic systems*) that embody

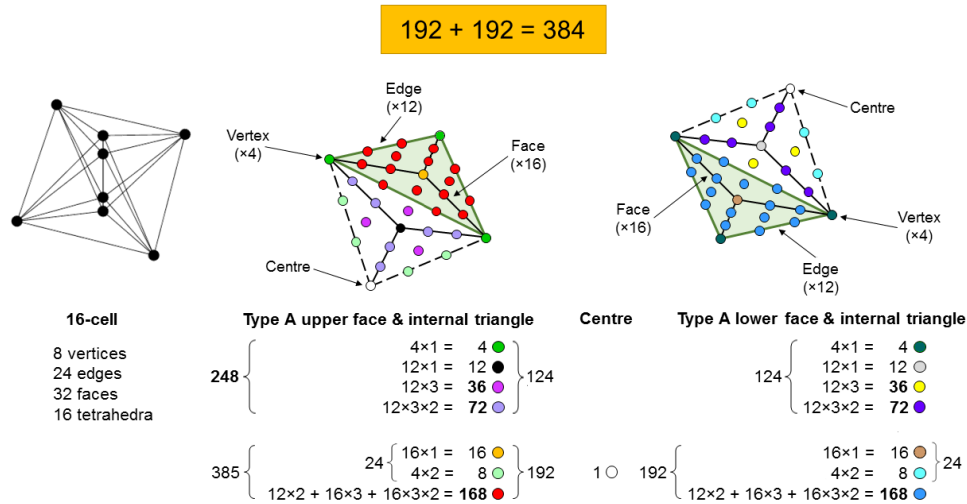


Figure 12

The 16-cell with Type A triangles has (**248+385=633**) yods

the divine archetypes, such as certain sacred geometries of the world's religions. *The numbers 137 and 496 appear in atomic and particle physics because they are signatures of holistic systems.* The problem of their appearance there exists only for the philosophies of materialism and atheism. They refuse to countenance the possibility of the transcendental as the ultimate source and cause of all things, and they have no room for the notion of "holistic" as they confuse what they deem irrational with what is beyond the bounds of science because it is *supra*-rational and the divine source of all rationality.

The identity:

$$384 + 248 = 632$$

has the following interpretation in terms of the various classes of yods that make up the 632 yods surrounding the centre of the 16-cell (see Fig. 12):

$$\begin{aligned} 384 &= [16 \text{ centres of faces} + 4 \times 2 \text{ hexagonal yods}] + 168 \text{ hexagonal yods in faces in one half} \\ &\quad + [16 \text{ centres of faces} + 4 \times 2 \text{ hexagonal yods}] + 168 \text{ hexagonal yods in faces in the other half} \\ &= (24+168) + (24+168) = 192 + 192. \end{aligned}$$

$$\begin{aligned} 248 &= 8 \text{ vertices} + 24 \times 3 \text{ hexagonal yods at centres of internal tetractyses} \\ &\quad + [24 \text{ centres of internal Type A triangles} + 24 \times 3 \times 2 \text{ hexagonal yods on sides of internal tetractyses}] \\ &= 8 + 72 + 168. \end{aligned}$$

We find that the classes of yods in the 16-cell naturally make up a set of 384 yods (192 yods in each half) whose distribution in 4-dimensional space conforms to the archetypal pattern of division of a holistic system (see here). We also find that the remaining **248** yods naturally divide into a set of eight yods corresponding to the eight simple roots of E_8 , a set of **72** hexagonal yods corresponding to the **72** roots of E_6 and a set of **168** yods corresponding to the remaining **168** roots of E_8 . There are too many correspondences to be attributable to chance. Its embodiment of the dimension **248** of E_8 , the dimension **496** of $E_8 \times E_8$ and the 192:192 pattern governing holistic systems found in sacred geometries amounts to convincing evidence for the holistic character of the 16-cell.

5. Non-coincidental, paranormally-derived numbers

Let us now examine the 16-cell composition of the compound of two 600-cells that is the E_8 Coxeter plane projection of the 4_{21} polytope. Each 600-cell is a compound of 5 24-cells, each 24-cell being a compound of three 16-cells. We found above that 56 yods line the 24 edges of a 16-cell when its faces are tetractyses. Hence, $(3 \times 56 = \mathbf{168})$ yods line the $(3 \times 24 = \mathbf{72})$ edges of three 16-cells in a 24-cell and 1680 yods line the 720 edges of the 30 16-cells in 10 24-cells (840 per 5 24-cells or **15** 16-cells in a 600-cell). *The number of boundary yods shaping all the 16-cells in two 600-cells is the very number of circular turns in each helical whorl of the UPA, which has an outer half comprising 840 turns and an inner half comprising 840 turns!* Can anyone believe that this is merely a coincidence? Well, perhaps one could do so with some justification if two 600-cells had no connection whatsoever to the symmetry groups describing superstring forces. But they *do* have a connection, for their compound is the E_8 Coxeter plane projection of the 240 vertices of the 4_{21} polytope, which mathematicians know represents the 240 roots of E_8 , the *very* symmetry group that appears (twice) in $E_8 \times E_8$ heterotic superstring theory! Moreover, the factorisation of 840 here is $5 \times \mathbf{168}$, where "5" denotes the 5 24-cells. Compare this with Leadbeater's account (summarised [here](#)), in which each whorl of the UPA revolves five times around its axis of spin, making 5 half-revolutions in its outer spiralling and five half-revolutions in its inner windings. It would seem that the natural conclusion is that the outer and inner halves of the UPA are the string manifestation of the E_8 gauge charges associated with the 120 vertices of each 600-cell, the five half-revolutions of the 10 whorls in each half being the string manifestation of gauge charges associated with five 24-cells. However, this cannot be the entire explanation, because the UPA has 16800 turns in its 10 whorls, whereas the number of boundary yods in the 30 16-cells is only 1680 (the number of turns in *one* whorl). The reader should understand that it is not being claimed here that this correlation amounts to the whole story, for the dynamic connection between circularly polarised, string-like oscillations and the yod composition of 16-cells has still to be elucidated. It is therefore more accurate to say that the 1680 boundary yods *correspond* to the 1680 turns that make up — not a whorl — *but a half-revolution of all 10 whorls of the UPA*. After all, its 10 whorls are what constitute the $E_8 \times E_8$ heterotic superstring, not a *single* whorl. Despite two 600-cells being a compound of 10 24-cells, a single 24-cell is *not* the basis in some still unexplained way for each whorl of the UPA. After all, the 4_{21} polytope merely represents through its 240 vertices the 240 roots of E_8 in an 8-dimensional space; it does not map as well the 3-dimensional *structure* of the UPA. Rather, because each whorl twists 10 times through an angle of 180° , every one of the 10 24-cells is associated with a rotation through a half-circle of all 10 whorls of the UPA, each one having the topology of a 1-torus. What is significant here is that, just as the E_8 Coxeter plane projection of the 4_{21} polytope is a compound of 10 24-cells, so, too, the 10 whorls of the UPA make 10 half-revolutions around its axis of spin. *Both systems are 10-fold because they are holistic objects*. In fact, the UPA is the realisation in 10-dimensional space-time of the E_8 -symmetric superstring force whose 240 gauge charges are mapped by the 240 vertices of the 4_{21} polytope.

The crucial point to be noted is that numbers like **168**, 336, 840, 1680, 3360 & 6720, which the author points out quantify what he claims are $E_8 \times E_8$ heterotic superstrings remote-viewed over a century ago, *naturally* appear among the properties of objects known to be mathematically connected to the symmetry group E_8 describing the unified force between these very superstrings! Moreover, they do so with a frequency that any reasonable person can see renders coincidence highly improbable. For the 16-cell, it is as obvious as its **168** triangles. Just how credible is it to attribute the appearance of just this number to chance, let alone all the other numbers? How, otherwise, can we account for the natural appearance in the 16-cell of *all* the Kabbalistic numbers except in terms of transcendental, mathematical design, as revealed through gematria in the sacred ancient Hebrew alphabet? There is no alternative, *realistic* explanation for how they quantify the properties of the 16-cell in such an uncontrived way. Such numbers appear for the simple reason that — like $E_8 \times E_8$ heterotic superstrings — the objects that embody them are all examples of *holistic systems* expressing the divine archetypes — a concept which, of course, most scientists deliberately *exclude* from consideration, although they do so for no better reason than that they do not accept as a *matter of principle* that such systems can exist. Let us be plain: there is only one *sensible* explanation for why certain numbers allegedly obtained by remote-viewing subatomic particles turn up in the 4_{21} polytope, E_8 and its exceptional subgroups so often as to make it highly improbable that this could happen by chance. It is that *the UPA described by Leadbeater really is an $E_8 \times E_8$ heterotic superstring*. It is as simple and as obvious as that! The alternative explanation requires all these numerous appearances of numbers supposedly obtained by paranormal means to be miraculous coincidences — a possibility which, statistically speaking, is so highly improbable that no one with any common sense would take it seriously. Is it any *more rational* or *more reasonable* to reject an ideologically problematic explanation, viz., UPAs are remote-viewed $E_8 \times E_8$ heterotic superstrings, in favour of one that is more acceptable to one's materialistic philosophy but requires having to believe that comments 1-11 listed

above reveal nothing more significant than the chance appearance of numerous Kabbalistic numbers? Why, for the sake of scientific propriety, should we have to accept that this absurd explanation is preferable when it is preferred by sceptics for no better reason than that it avoids their having to accept that the paranormal ability to remote-view subatomic particles exists because it was successfully demonstrated by the Theosophists Annie Besant and C.W. Leadbeater? Why should a personal ideology trump common sense as to what can be plausibly attributed to coincidence?! Well, of course, it should not do so. But it *will* for materialists who, faced with evidence that they cannot deny, desperately try to salvage their cherished philosophy by any means they can find, whether it is credible or not. However, a more sensible person unbiassed by philosophical viewpoints such as atheism or materialism will see the appeal by sceptics to coincidence for what it clearly is, viz., blatant confirmation bias exhibited by those who habitually refuse to recognise *any* evidence — however strong — that discredits their ideological aversion towards the paranormal and the transcendental and who resort to implausible explanations requiring multiple miracles of chance to occur in order to provide themselves with *any* excuse — however flimsy — for rejecting this evidence. In science, *prima facie* evidence for a phenomenon — *whatever the implications of its existence* — must always trump personal presuppositions about what is, and what is not, possible, otherwise its assessment ceases to be scientific and becomes subjective. But that, of course, is not how every scientist thinks....
