Article 63

The Sacred Geometrical Nature of the 4_{21} Polytope

by

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Abstract

It is proved that the properties of various sacred geometries embody the number (6720) of edges of the 4_{21} polytope and the number (240) of its vertices, which define the 240 roots of the exceptional Lie group E_8 that determines one of the two symmetry groups for heterotic superstrings. The number 6720 is implicit in the paranormal description of superstrings published over a century ago by the Theosophists Annie Besant and C.W. Leadbeater.

For more details, see "4-d sacred geometries" at:

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Commentary on Fig. 1

The number of faces of the 4_{21} polytope = 60480 = 9×6720. The number of edges = 6720. The number of edges & faces = 6720 + 9×6720 = 10×6720 = 67200 = **50**×1344. In other words, it is the number of yods intrinsic to the 700 Type B polygons enfolded in **50** overlapping Trees that surround their centres.* This shows how ELOHIM, the Godname of Binah with number value **50**, prescribes the number of edges & faces in the 4_{21} polytope. Joining its 240 vertices to its centre generates 6720 internal triangles with 240 internal sides. The number of geometrical elements surrounding the centre of the polytope = 240 + 67200 + 240 + 6720 = 74400 = **31**×2400 = **620**×120, where **620** is the number value of Kether. This is a beautiful property because the number of hexagonal yods in a ten-sided decagon with 2nd-order tetractyses as sectors is **620**, whilst 120 = $11^2 - 1$ = 3 + 5 + 7 +... + **21**, i.e., 120 is the sum of the first ten odd integers after 1. Assigning 120 to each hexagonal yod in this decagon generates the minimum number of geometrical elements needed to construct the interior and faces of the 4_{21} polytope. The Pythagorean Decad arithmetically determines the geometrical composition of the very polytope that represents the 240 roots of the exceptional Lie group E_8 used in $E_8×E_8$ heterotic superstring theory. Notice in the table that the numbers of sides and triangles are integer multiples of the number of vertices (240). In particular, the number of triangles = **280**×240, where **280** is the number of *Sandalphon*, the Archangel of Malkuth. This is also the number of edges & faces. The total number of corners, sides & triangles = 310×240 = **31**×2400, where **31** is the number of EL, the Godname of Chesed, so that the number of such geometrical elements in each half of the polytope = **155**×240, where **155** is the number value of ADONAI MELEKH, the complete Godname of Malkuth. We see that the gematria numbers of both the Godname and Archangel of Malkuth, the physical manifestation of the Tree of Life blueprint, are present in the geometrical composition of the 4_{21} polytope. This, of course, is not unexpected, given its fundamental, mathematical status in representing the roots of the Lie symmetry group E_8 describing the forces between one of the two types of heterotic superstrings — the microphysical realisation of this blueprint.

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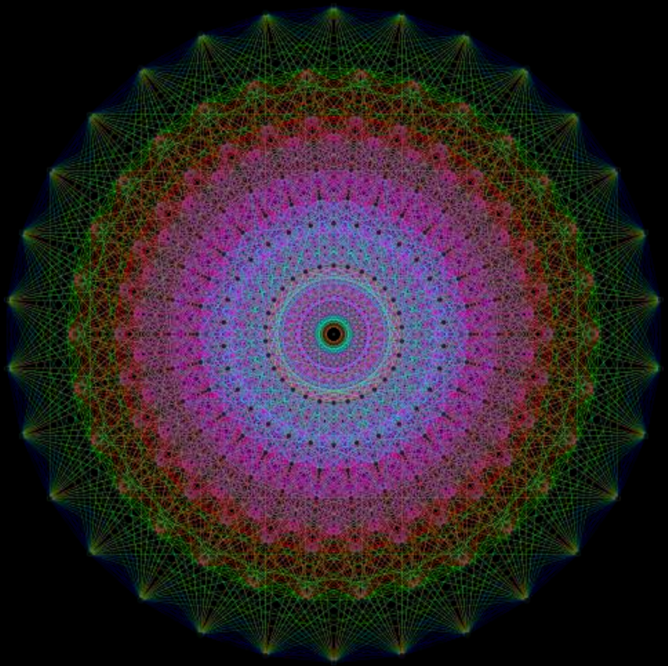
* Proof: The (7+7) enfolded Type B polygons have 1370 yods. Therefore, the (7n+7n) Type B polygons enfolded in n Trees have (1368n+2) yods. Seven yods line the vertical axis of each hexagon. They are shared with the outer Tree of Life. Number of shared yods in the (7n+7n) polygons = 6n + 1 + 6n + 1 = 12n + 2. They contain the centres of all the hexagons. Number of centres of the remaining 12n polygons = 6n + 6n = 12n. Number of yods either centres or shared with the n Trees of Life = 12n + 2 + 12n = 24n + 2. Number of yods intrinsic to (7n+7n) polygons and surrounding their centres = 1368n + 2 – (24n+2) = 1344n. For n = **50**, this is **50×**1344 = 67200.

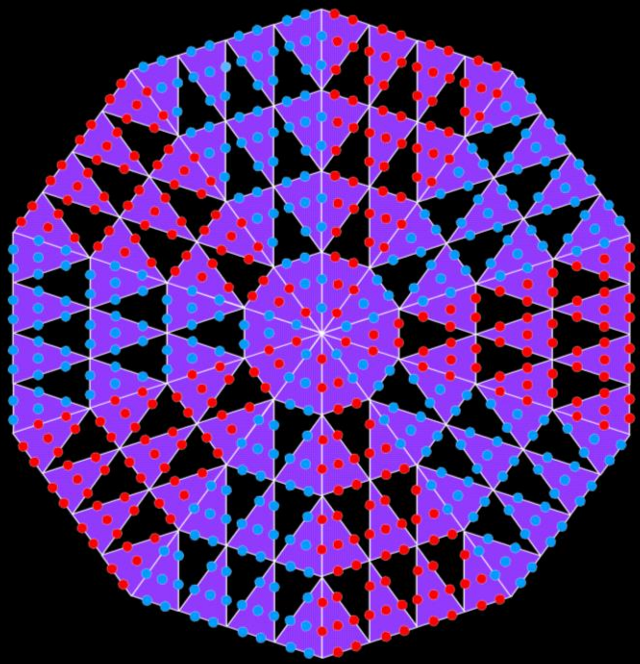
Figure 1

The geometrical composition of the faces & interior of the 4_{21} polytope constructed from triangles

	Corners	**Sides**	**Triangles**	**Total**
Faces	240	6720 = 28×240	60480 = 252×240	67440 = 281×240
Interior	0	240	6720 = 28×240	6960 = 29×240
Total	240	6960 = 29×240	67200 = **280**×240	74400 = 310×240



4_{21} polytope



74400 = **620**×120

= = 120 =

3

5 7

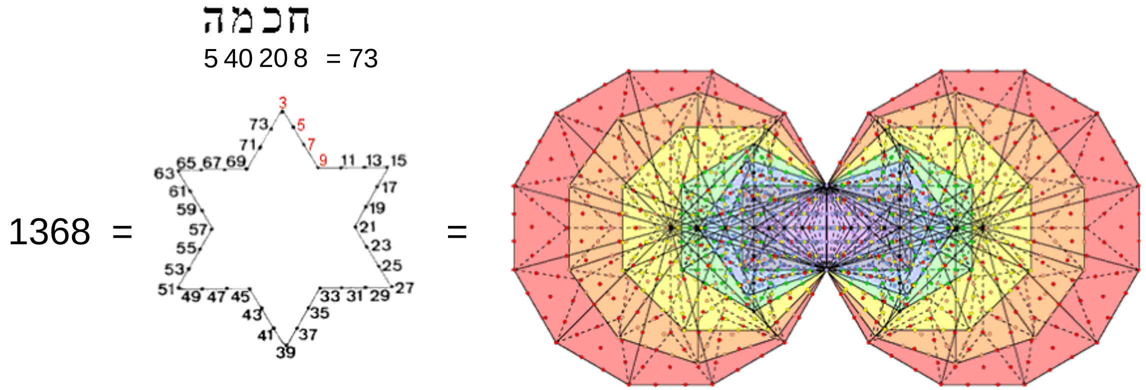
9 11 13

15 17 19 **21**

The Decad/tetractys determines the number of points, lines & triangles in the 4_{21} polytope.

Commentary on Fig. 2

As 672 = 4×**168** = 8×84 and 24 = 8×3, the number of vertices in the 4_{21} polytope = 240 = **80**×3 and its number of edges = 6720 = **80**×84, so that its number of vertices & edges = **80**×(3+84) = 6960 = **80**×**87**. Remarkably, **80** is the number of Yesod and **87** is the number value of *Levanah*, its Mundane Chakra! Suppose that its 60480 triangular faces are tetractyses (ordinary, Type A, Type B, etc.). The number of hexagonal yods lining their sides = 2×6720 = 13440 = **80**×**168**, where **168** is the number value of *Cholem Yesodoth*, the Mundane Chakra of Malkuth. Given that Malkuth follows Yesod in the Tree of Life, this is equally remarkable. The number of yods lining the edges of the 4_{21} polytope constructed from tetractyses = 240 + 2×6720 = 13680 = **80**×3 + **80**×**168** = **80**×171 = 10×**36**×38 = 10(37^2 − 1) = 10×(3 + 5 + 7 +... + **73**), i.e., the Godname ELOHA of Geburah with number value **36** quantifies the *shape* of the polytope because 13680 is ten times the sum of the first **36** odd integers after 1 up to **73**, which is the number of Chokmah:



3 + 5 + 7 + 9 = 24 → 24 (●) centres or yods shared with outer Tree of Life

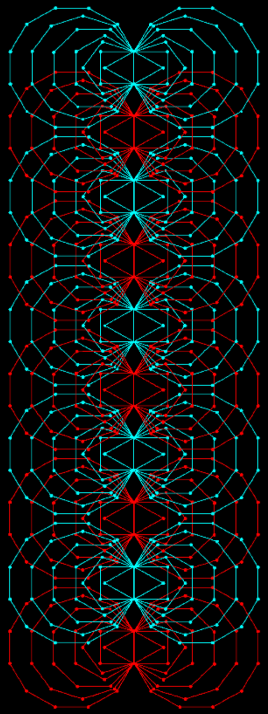
11 + 13 + …+ **73** = 1344 → 1344 intrinsic yods surround centres of 14 polygons

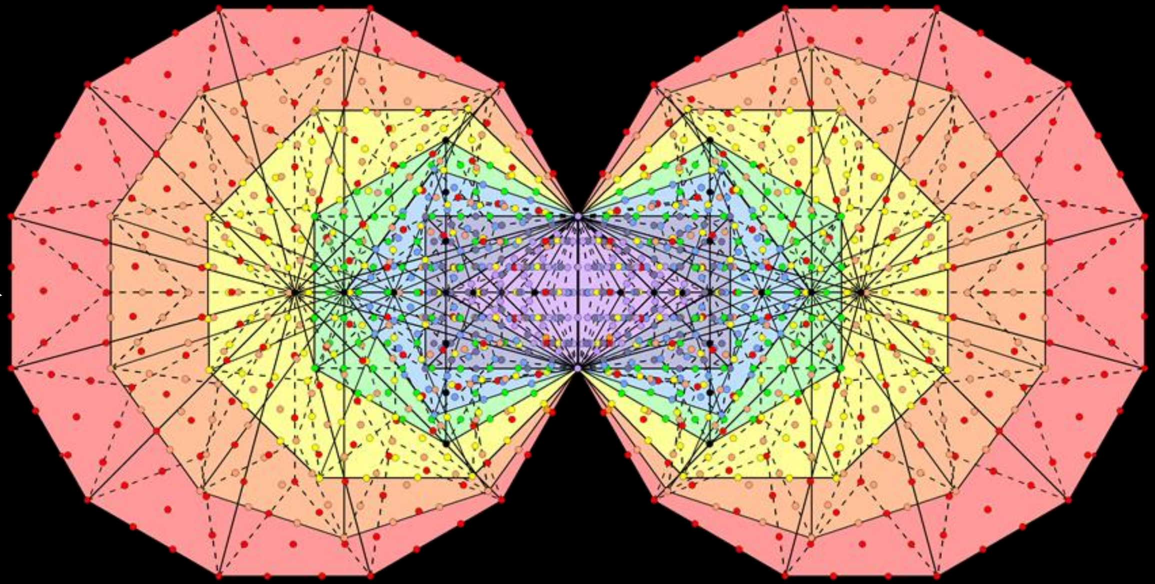
The number value 73 of Chokmah and the number value 36 of ELOAH, the Godname of Geburah, determine the 1368 yods intrinsic to the (7+7) enfolded Type B polygons of the inner Tree of Life as the sum of the first 36 odd integers after 1.

As **36** yods line the boundary of a Star of David, assigning these **36** odd integers to them generates the number 1368, which is the number of yods intrinsic to the (7+7) enfolded Type B polygons (see diagram above). As 24 = 3 + 5 + 7 + 9, the first four odd integers (written above in red) sum to the number of black centres or yods that are shared with the outer Tree of Life; they form one side of the Star of David. The next 32 odd integers add up to 1344, which is the number of yods unshared with the outer Tree of Life that surround centres of the 14 polygons. The 240 centres or yods shared with 10 Trees of Life denote the 240 vertices of the 4_{21} polytope and the 13440 yods surrounding the centres of the **140** polygons enfolded in 10 Trees correspond to the 13440 hexagonal yods lining the 6720 edges of the polytope.

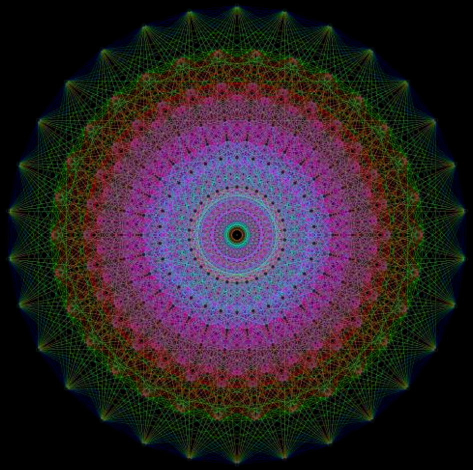
Figure 2

(70+70) polygons enfolded
in 10 Trees of Life





4_{21} polytope



26 black yods are either unshared centres of polygons (10) or shared with the outer Tree of Life (16) because they either lie on the vertical axes of the two hexagons that coincide with the two side pillars or are centres of the two triangles that coincide with the two hexagonal yods on the Chesed-Geburah Path. The (7+7) enfolded Type B polygons have 1370 yods (1368 yods per set of 14 polygons). Each set of 7 enfolded polygons has 672 intrinsic yods surrounding their centres.

4_{21} polytope	Inner form of 10 Trees of Life
240 vertices & 6720 edges of 60480 triangular faces;	240 black yods that are either centres of polygons or shared with outer form of 10 Trees;
(240 + 2×6720 = 13680) yods line edges of 4_{21} polytope with tetractyses as its faces.	13680 yods intrinsic to the (70+70) Type B polygons enfolded in 10 Trees of Life. They comprise 240 centres or shared yods and 6720 yods surrounding centres of each set of 70 polygons.

Correspondence between the 4_{21} polytope and the inner form of 10 Trees of Life.

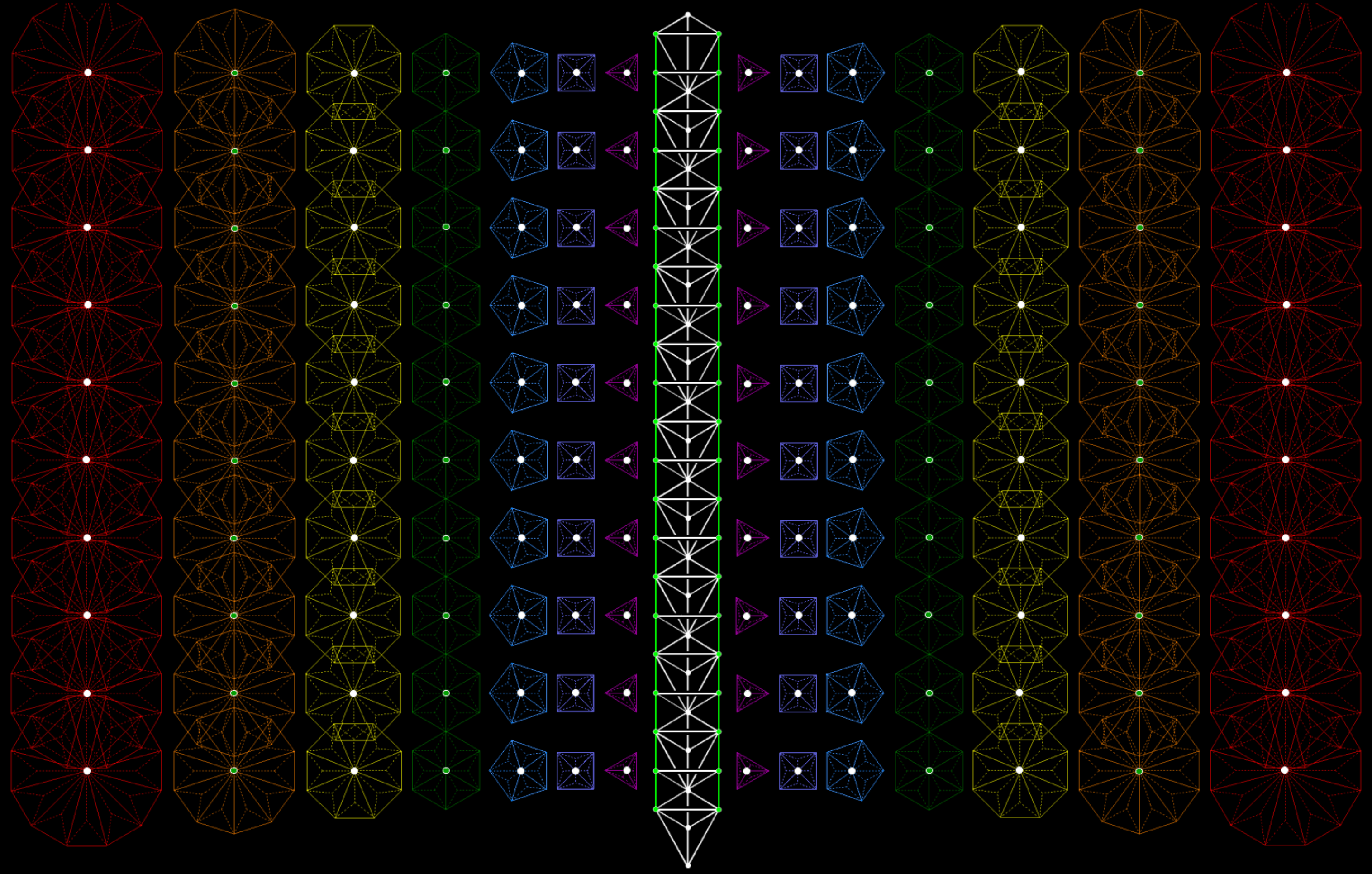
Commentary on Fig. 3

When the faces of the 4_{21} polytope are tetractyses, there are (3360×2=6720) hexagonal yods on the 3360 edges in each half of it. We saw earlier that this is the number of yods surrounding the centres of the set of seven enfolded Type B polygons enfolded in each one of 10 Trees of Life that are unshared with them. It will now be shown that the same property applies to these polygons when separate. A Type B n-gon contains (**15**n+1) yods, where "1" denotes the yod at its centre. Given an n-gon divided into its sectors,14n more yods are needed to turn its sectors into Type A triangles. The seven separate Type B polygons have **48** sectors. Their transformation into Type A triangles requires (14×**48**=672) more yods. 6720 more yods are needed to turn into Type B polygons the 70 polygons in each half of the inner form of 10 Trees of Life when they are regarded as separate. The centre of the enfolded triangle and the seven yods on the vertical axis of the enfolded hexagon coincide with yods in triangles belonging to the outer Tree of Life. The number of centres of each set of seven separate polygons and shared yods = 1 + 7 + 5 = 13. One of them (Chokmah) is Netzach of the next higher Tree. Therefore, the number of shared yods or centres in n Trees and their 14n separate polygons = 12n + 1 + 12n + 1 = 24n + 2. This means that there are 240 shared yods or centres that are intrinsic to 10 overlapping Trees because they are not shared with the 11th Tree. They are the counterpart of the 240 vertices of the 4_{21} polytope. The shared outer form of 10 Trees of Life and their inner form as separate polygons represent the 13680 yods that are needed to create the edges of the 4_{21} polytope. As we saw earlier, the inner form of 10 Trees by itself embodies this number as the yod population of their enfolded polygons. The crucial number to be accounted for here is the number 6720, namely, the number of edges of the 4_{21} polytope. It emerges naturally as:

- the number of yods that are intrinsic to the inner form of 10 Trees of Life and which surround centres of enfolded polygons;
- the number of yods needed to turn into Type triangles all the **48** sectors of the 70 separate polygons present in the inner form of 10 Trees of Life: 10×**48**×14 = 6720.

In the second case, a pair of hexagonal yods on each edge of the polytope corresponds to a yod in one set of 70 polygons and its counterpart in the mirror image set of 70 polygons on the other side of the central pillar of the 10 Trees. Just as the 6720 edges give shape to the polytope whose vertices are 240 points distributed in 8-dimensional space, so the inner form of 10 Trees of Life is given form by 6720 yods other than the 240 centres of polygons and the yods they share with the outer form of these Trees. It cannot be a coincidence that the number of edges of such a physically significant object like the 4_{21} polytope is equal to the number of plane-polarised waves that make up the 3360 circularly polarised oscillations in one revolution of the 10 whorls of the UPA, the basic constituent of atomic nuclei remote-viewed by Annie Besant & C.W. Leadbeater over a century ago.

Figure 3



240 white centres or corners & sides of triangles unshared with inner form of 10 Trees of Life

3360 corners & sides surround 70 centres of polygons

3360 corners & sides surround 70 centres of polygons

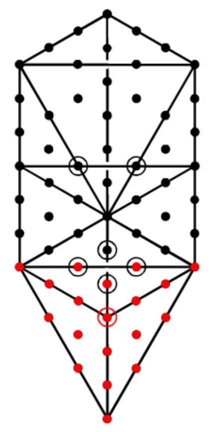
3360 edges in half of 4_{21} polytope

240 vertices in 4_{21} polytope

3360 edges in half of 4_{21} polytope

(Green centres are excluded from the counting of centres because they become corners of enfolded polygons, which are counted)

The five corners & sides on each side pillar of the outer Tree of Life are shared with the two hexagons in the inner Tree of Life because these pillars are their vertical axes. This means that (8n+2) corners & sides in n overlapping Trees are shared with the 2n hexagons in their inner form. Any n overlapping Trees of Life have (12n+4) triangles with (6n+4) corners and (16n+6) sides, i.e., (22n+10) corners & sides. Of these, (8n+2) corners & sides are shared, leaving (14n+8) corners & sides that are unshared. 10 overlapping Trees have **148** corners & sides of 124 triangles that are intrinsic to these Trees. **148** is the number value of Netzach. Every 10 Trees have (14×10=**140**) corners & sides of 120 triangles that are unshared with the **140** polygons making up their inner form, where **140** is the number of *Malachim*, the Order of Angels assigned to Tiphareth. Every 10 Trees have (**140**+**140**=280) unshared corners & sides or centres of separate polygons, where **280** is the number value of *Sandalphon*, the Archangel of Malkuth. Two of the centres of the seven polygons enfolded in each Tree are corners of the latter, so that five centres do not become corners as well when the separate polygons become enfolded. The 70 centres of the 10 sets of seven polygons on either side of 10 Trees consist of 20 centres that become corners and **50** centres that remain just centres. Hence, 100 of the **140** centres of the **140** separate polygons remain just as centres when they are enfolded. 10 overlapping Trees and the **140** separate polygons have (**148**+100=**248**) unshared corners & sides belonging to the former or centres of the latter that do not become corners when they are enfolded. **248** is the number value of *Raziel*, the Archangel of Chokmah. For every 10 Trees, there are **140** unshared corners & sides and 100 pure centres, that is, 240 geometrical elements, which either belong solely to the outer form of 10 Trees or are centres of the polygons making up their inner form that remain just centres when they become enfolded. They comprise 120 corners/centres and 120 sides. 3360 corners & sides of triangles surround the centres of the 70 separate polygons on either side of 10 overlapping Trees. So we discover that in every 10 Trees, 6720 corners & sides of 2820 (=10×**282**) triangles surround the centres of polygons, where **282** is the number value of *Aralim*, the Order of Angels assigned to Binah. This leaves 240 corners & sides that do not appear among the former when the polygons become enfolded (the centres of the hexagon & decagon in each set of seven polygons become corners of the triangle and pentagon, which are counted amongst the 336 corners & sides that surround the seven centres — hence, they are coloured green to indicate their exclusion from the count). The 240 corners & sides comprise 120 unshared corners & pure centres of polygons and 120 unshared sides of triangles in 10 Trees of Life.

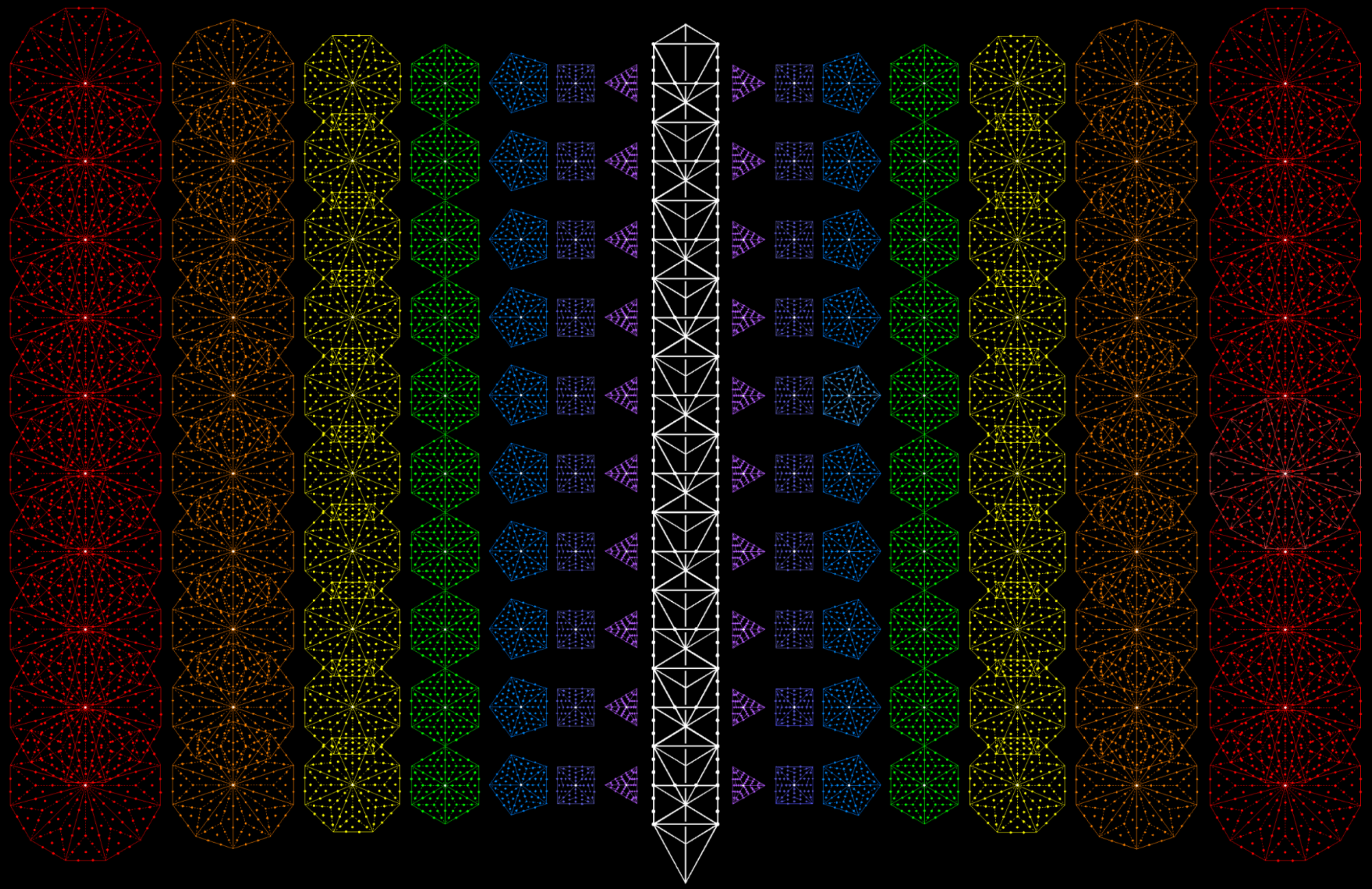


70 = 50 (●) + 20 (●)

Compare the appearance of the numbers 240 and 6720, which measure the numbers of corners & sides in the outer and inner form of every 10 Trees, with the 240 vertices and 6720 edges of the 4_{21} polytope. The fact that the latter number manifests in 10 Trees of Life demonstrates the Tree of Life nature of this polytope because this number of Trees represents the 10 Sephiroth of a single Tree. Their counterpart in the UPA are the 240 gauge charges of E_8 and the 3360 turns in each revolution of its 10 whorls, each turn being a circularly polarised wave that is a supposition of two perpendicular plane waves with a phase difference of 90°. In other words, the 4_{21} polytope represents through its vertices not only the roots of E_8 but also through its 6720 edges the form of the E_8×E_8 heterotic superstring (UPA), each edge corresponding to a plane wave component of the 3360 circularly polarised waves that make up one revolution of its 10 whorls.

There is a 20:**50** division of the 70 polygons enfolded on either side of the central Pillar of Equilibrium of 10 Trees of Life according to whether their centres become corners (20) or not (**50**) when they become enfolded. This pattern exists in the distribution of the 70 yods in the 16 tetractyses of the outer Tree of Life into 20 yods that belong to the tetrahedron at its base and **50** yods that make up the remaining 12 tetractyses. This division manifests in the 4_{12} polytope as the 120 vertices and 3360 edges in each half because their counterparts in the outer and inner form of 10 Trees of Life are the 120 unshared corners & pure centres and 120 unshared sides in the outer form of 10 Trees and the 3360 corners & sides that surround the centres of the 70 polygons enfolded on either side of them. Here, once more, is the 120:120 division displayed by holistic systems that embody the number 240. It manifests as the compound of two 600-cells, each with 120 vertices, that is the 4-dimensional, Coxeter plane projection of the 4_{21} polytope.

Figure 4



6720 yods other than corners of polygons surround 70 white centres of polygons

↓

6720 hexagonal yods on 3360 edges in half of 4_{21} polytope

240 white centres or yods in outer form of 10 Trees of Life shared with their inner form

↓

240 vertices in 4_{21} polytope

6720 yods other than corners of polygons surround 70 white centres of polygons

↓

6720 hexagonal yods on 3360 edges in half of 4_{21} polytope

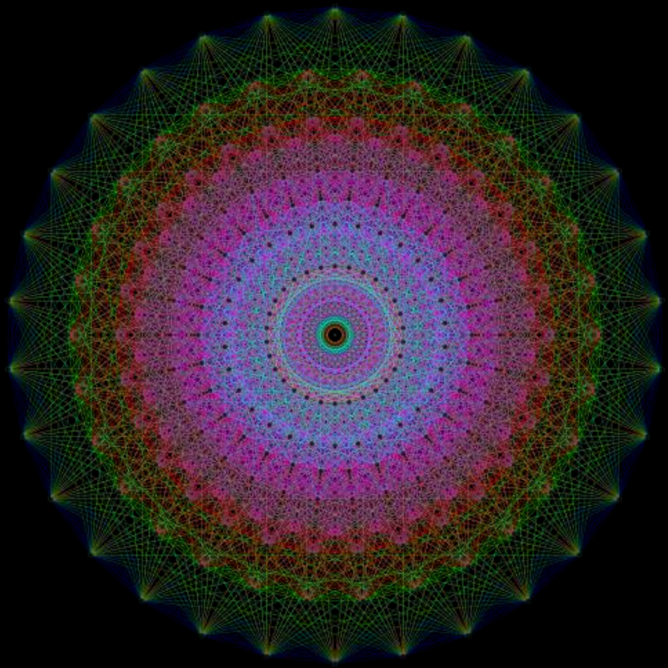
Commentary on Fig. 5

A Type A triangle contains 19 yods. Of these, three are corners, so that 16 more yods are needed to turn a simple triangle into a Type A triangle. The 42 triangles surrounding the centre of the 2-dimensional Sri Yantra contain (42×16=672) yods other than their 68 corners when they are Type A triangles. In other words, 672 more yods are needed to construct the 42 triangles from tetractyses. When each yod is weighted with the number 10, the Sri Yantra embodies the number 6720. This is the number of edges in the 4_{21} polytope.

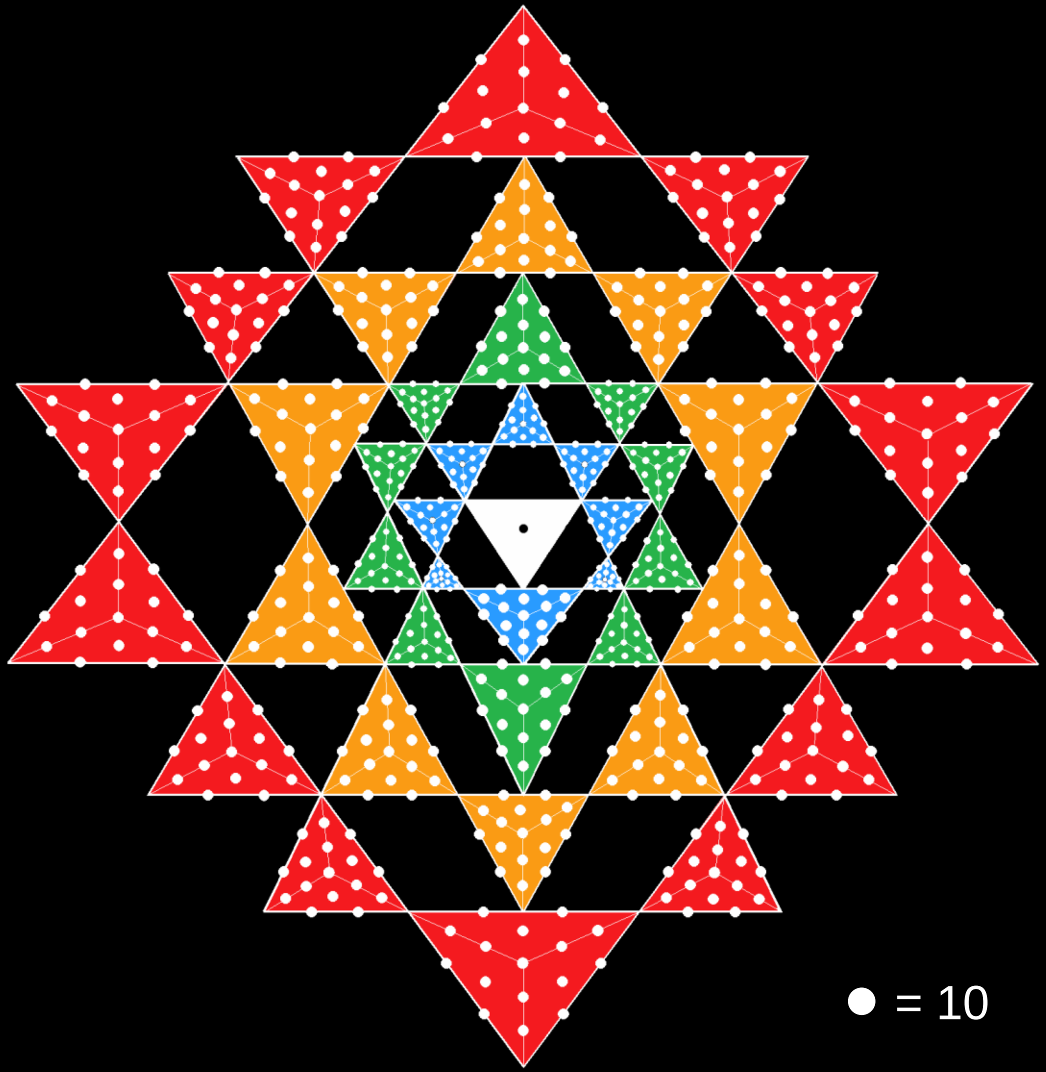
Although traditionally regarded as a triangle, the downward-pointing, three-sided figure at the centre of the Sri Yantra is not, from a strict, mathematical point of view, a triangular area because the bindu symbolising the Absolute is assigned to its centre. For this reason, it is not legitimate to treat this figure as a Type A triangle on the same par as the 42 triangles that surround it.

Of the 672 yods in the 42 Type A triangles, 42 are corners of tetractyses, leaving **630** hexagonal yods. The number **630** is the number value of *Seraphim*, the Order of Angels assigned to Geburah.

Figure 5



The 4_{21} polytope has 6720 edges



Weighted with the Decad (10), the 672 yods in the 42 Type A triangles of the Sri Yantra that are not their corners generate the number of edges of the 4_{21} polytope.

Commentary on Fig. 6

Suppose that the 38 faces of the first four Platonic solids are constructed from Type A polygons. The 90 internal triangles formed by joining their vertices to their centres may be regarded as simple tetractyses or as Type A triangles. For the present purpose, only the former will be discussed here. The table shown below lists the yod populations of their faces and interiors:

Number of yods in the faces & interiors of the first four Platonic solids

	V	E	F	Faces			Interior					
				Number of corners	Number of hexagonal yods	Total	Number of corners	Number of hexagonal yods	Total	Number of corners	Number of hexagonal yods	Total
Tetrahedron	4	6	4	8	**48**	56	1	14	14+1	8+1	**62**	70+1=71
Octahedron	6	12	8	14	96	110	1	24	24+1	14+1	120	134+1=135
Cube	8	12	6	14	96	110	1	28	28+1	14+1	124	138+1=139
Icosahedron	12	30	20	32	240	**272**	1	54	54+1	32+1	294	326+1=327
Total	30	60	38	68	480	548	1	120	120+4	68+4	600	668+4=672

("1" denotes the centre of a Platonic solid)

Including their centres, the tetrahedron, octahedron, cube & icosahedron have 672 yods (124 internal, 548 in faces). On average, they have (672/4=**168**) yods made up of (124/4=**31**) internal yods and (548/4=137) yods in faces. Amazingly, the first four Platonic solids embody on average both the superstring structural parameter **168** and the number 137, one of the most important numbers in physics because its reciprocal measures the approximate value of the fine-structure constant! This number is the difference between **168**, which is the number value of the Mundane Chakra of Malkuth, and **31**, which is the number value of EL ("God"), the Godname of Chesed:

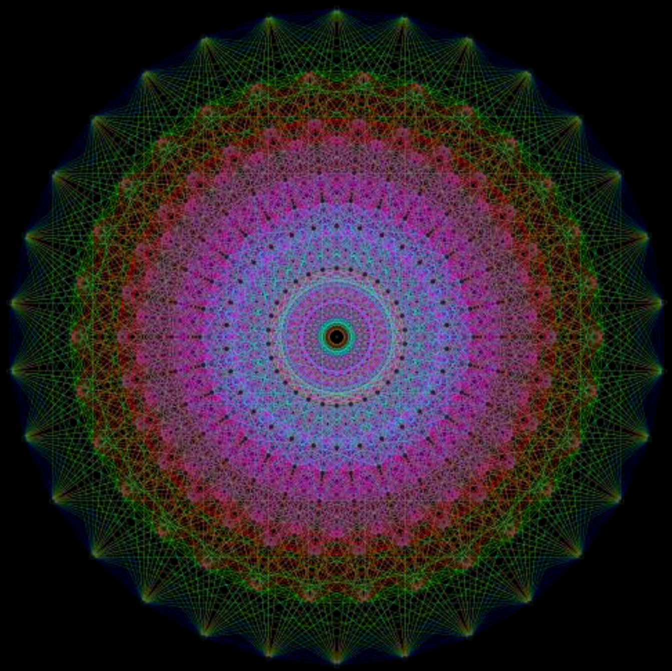
$$137 = \mathbf{168} - \mathbf{31}.$$

137 is the average number of yods making up the faces of the first four Platonic solids when constructed from Type A triangles.

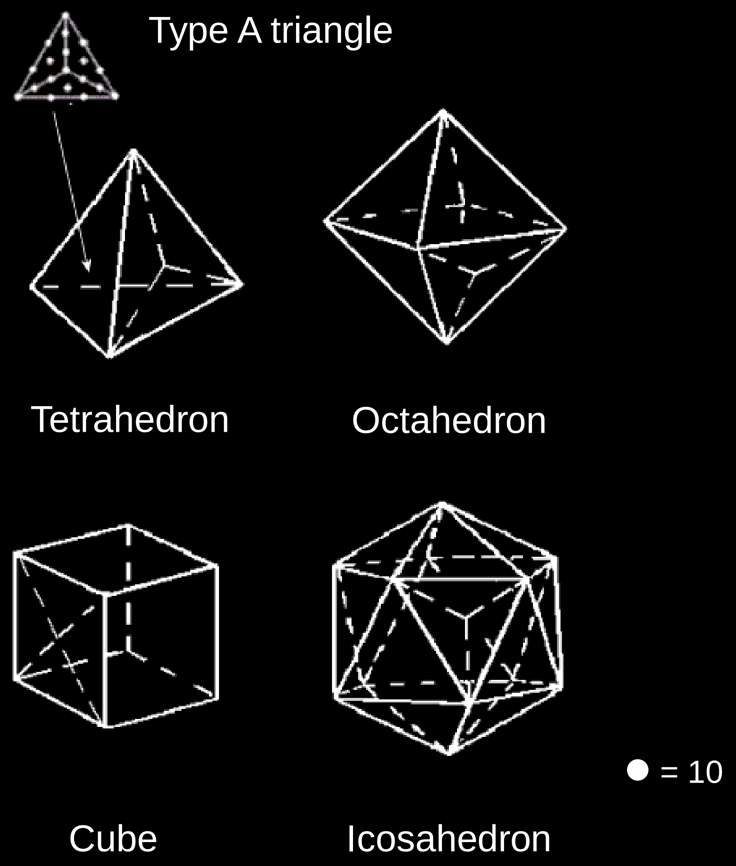
Now suppose that each of the 672 yods is weighted with the number 10. The first four Platonic solids then embody the number 6720. This is the number of edges in the 4_{21} polytope. The average of the numbers generated by their weighted yods is 1680. This is the number of turns in each helical whorl of the UPA. It is the sum of the number 310 generated by their internal yods and the number 1370 generated by the yods in their faces. The latter number is the yod population of the inner Tree of Life with Type B polygons. In the discussion of the 24-cell in 4-dimensional sacred geometries, we found that its faces have 672 corners, lines & triangles in addition to its vertices & edges, when constructed from Type A triangles. In the discussion in the commentary on Fig. 2 of the arithmetic properties of the 4_{21} polytope, we pointed out that, associated with each set of seven enfolded Type B polygons, there are 672 intrinsic yods surrounding their centres. These are the yods belonging solely to each set that actually create its form. The number 672 performs the same role in the context of the first four Platonic solids, only this time it quantifies the yods needed to create the analogous, holistic system in three, not two, dimensions of space. For the 24-cell, it determines its faces in four-dimensional space. Multiplied by the Decad, the number 672 quantified the edges that shape the faces of the 4_{21} polytope in eight-dimensional space. In the case of the UPA/subquark superstring, it is the number of perpendicular plane waves differing in phase by 90° whose supposition creates 336 circular turns in each revolution of a whorl around the axis of spin of the UPA.

Figure 6

4_{21} polytope



6720 =

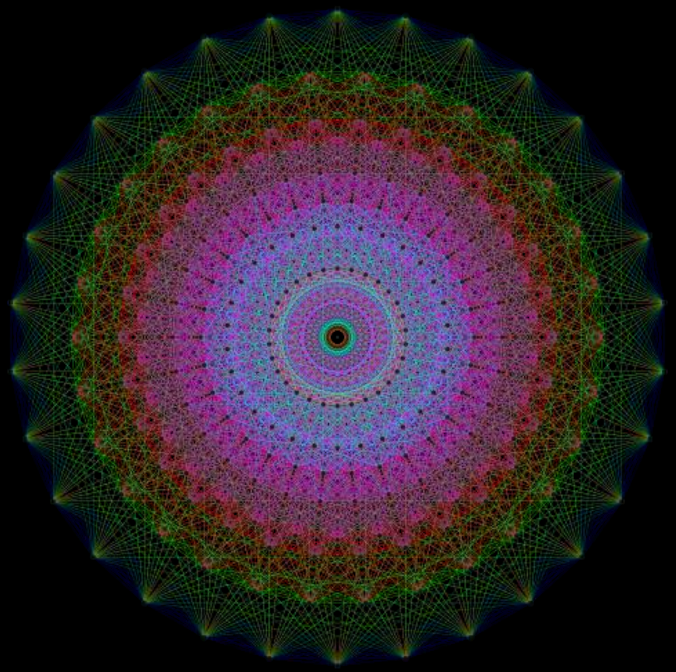


Weighted with the Decad, the 672 yods in the first four Platonic solids constructed from Type A triangles generate the number of edges of the 4_{21} polytope.

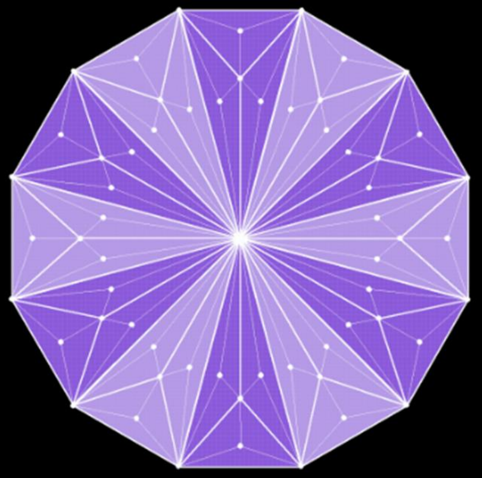
Commentary on Fig. 7

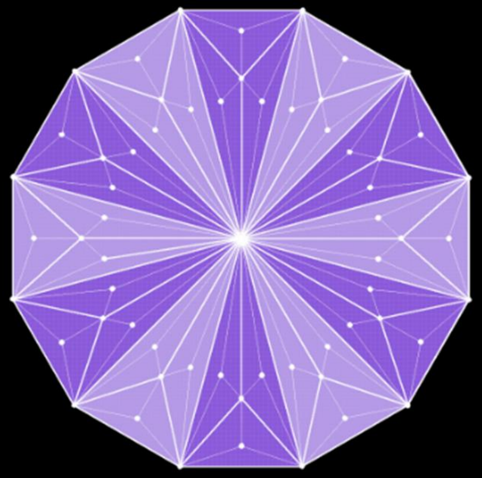
A Type C polygon has Type B triangles as its sectors. Apart from their shared centre, there are five corners, 14 sides & nine triangles per sector, i.e., 28 geometrical elements. Surrounding the centre of the Type C dodecagon are (12×28=336) geometrical elements. They comprise [12×(5+9)=**168**] corners & triangles and (12×14=**168**) sides. This **168**:**168** division is characteristic of holistic systems displaying this parameter, e.g., the 336 yods lining the 126 sides of the 42 triangles surrounding the centre of the 3-dimensional Sri Yantra consist of 168 yods on the sides of the 21 triangles in each half of it. (336+336=672) geometrical elements surround the centres of the two separate Type C dodecagons as 336 sides and 336 corners & triangles. This property is the counterpart of the 672 yods intrinsic to each set of seven enfolded Type B polygons and surrounding their centres that was discussed in the commentary on Fig. 2. It is also the counterpart of other sacred geometries previously analysed. However, unlike yods, the number 10 cannot be meaningfully assigned to lines and triangles. Instead, we must remember that — as considered earlier for the Type B polygons — 10 overlapping Trees of Life can represent the single Tree composed of 10 Sephiroth. Surrounding the centres of the 20 separate, Type C dodecagons that are part of their inner form are 6720 geometrical elements (3360 sides, 3360 corners & triangles). As a holistic system in itself, the pair of Type C dodecagons through the 10-fold Tree of Life generates the number 6720. This is the number of edges in the 4_{21} polytope whose vertices determine the roots of E_8, which is the exceptional Lie group that describes the unified symmetry of superstrings whose 10 dimensions are mapped by these 10 Trees of Life.

Figure 7



Each half of the 4_{21} polytope has 3360 edges





Constructed from Type B triangles, the two Type C dodecagons have 672 corners, sides & triangles surrounding their centres. The 20 such dodecagons in the inner form of 10 Trees of Life have 6720 geometrical elements surrounding their centres. They correspond to the 6720 edges of the 4_{21} polytope.

Commentary on Fig. 8

A Type A triangle contains 19 yods. Nine yods line its sides and 10 yods (one corner & nine hexagonal yods in three tetractyses) lie inside its boundary. A Type B triangle contains 46 yods. Nine yods line its sides and 37 yods (four corners and 33 hexagonal yods in nine tetractyses) are internal.

The disdyakis triacontahedron has **62** vertices (60 surrounding an axis passing through two diametrically opposite vertices), 180 edges & 120 triangular faces. When all faces and internal triangles formed by joining vertices to its centre are Type A:

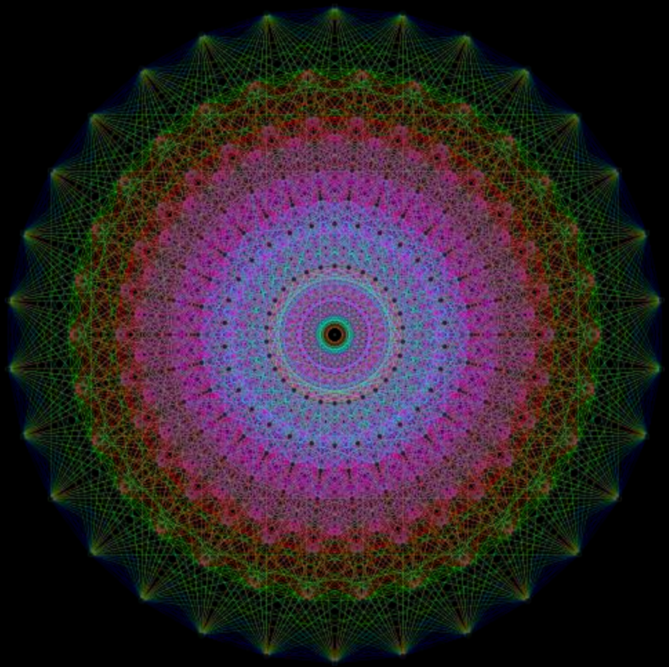
Faces: The 120 Type A triangles have (120×3=360) tetractyses with 120 internal corners and (120×9=1080) internal hexagonal yods. They have 60 external corners surrounding an axis passing through two diametrically opposite vertices.

Edges: (180×2=360) hexagonal yods line 180 edges.

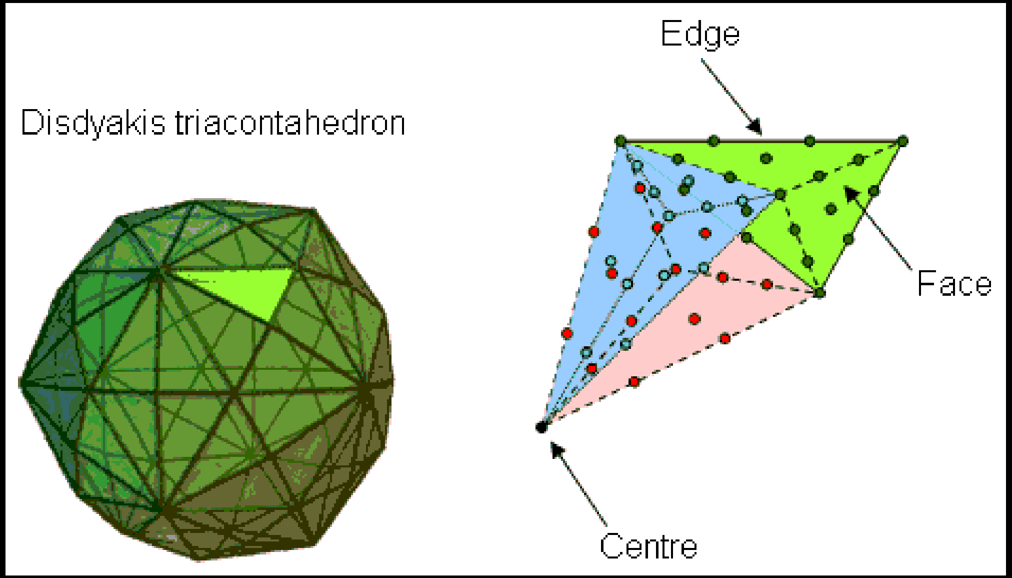
Interior: The 180 edges extend 180 Type A triangles with 180 internal corners of (180×3=540) tetractyses and (180×9=1620) hexagonal yods. Their internal sides have (60×2=120) hexagonal yods. The (120×3=360) sides of tetractyses in the faces extend 360 Type A triangles with 360 internal corners of (360×3=1080) tetractyses and (360×9=3240) hexagonal yods. Their sides have (120×2=240) hexagonal yods as well as the 120 hexagonal yods generated by sides joining the centre of the polyhedron to the 60 vertices surrounding its axis.

The number of corners = 60 + 120 + 180 + 360 = 720. The number of hexagonal yods = 1080 + 360 + 1620 + 120 + 3240 + 240 = 6660. The total number of yods surrounding the axis = 720 + 6660 = 7380. Therefore, (6660+60=6720) yods other than new corners of tetractyses are needed to construct the disdyakis triacontahedron. This is how the disdyakis triacontahedron embodies the holistic parameter 6720 that manifests in the 4_{21} polytope as its 6720 edges.

Figure 8



The 4_{21} polytope has 6720 edges



6720 yods other than new corners are needed to construct the faces & interior of the disdyakis triacontahedron from Type A triangles (the three types are coloured green, pink & blue).

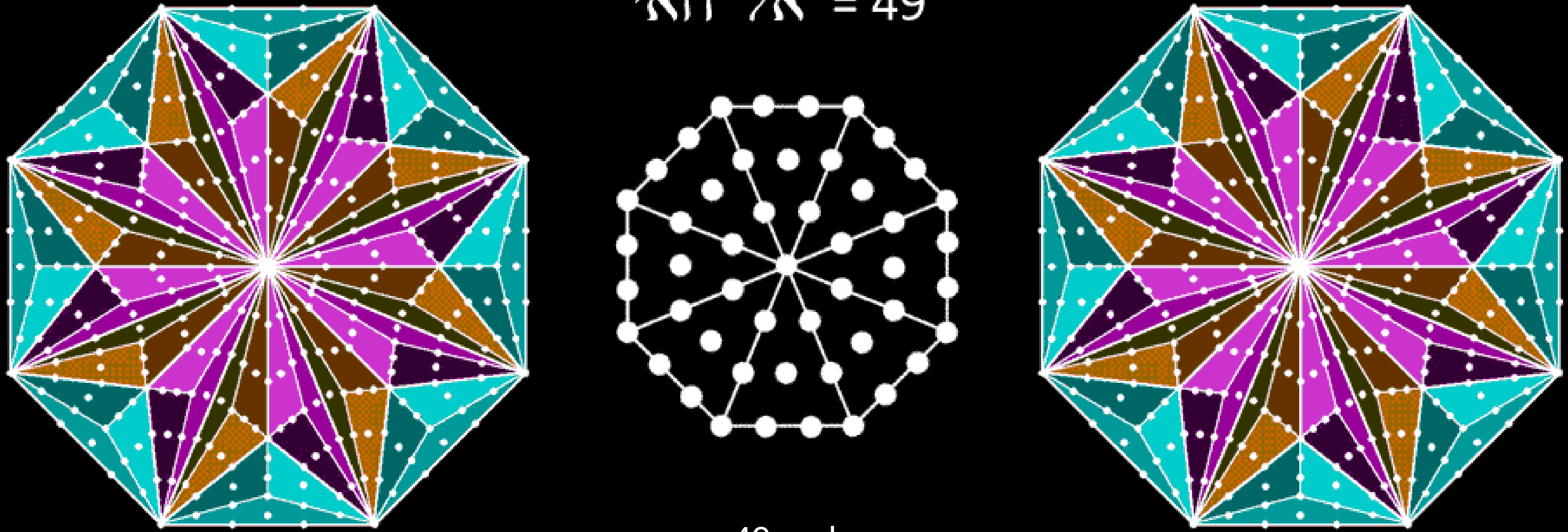
Constructed from Type A triangles, the disdyakis triacontahedron is composed of as many yods other than new corners as the 4_{21} polytope has edges.

Commentary on Fig. 9

An octagon whose sectors are tetractyses contains **49** yods. This is the number value of EL CHAI ("God Almighty"), the Godname of Yesod. The centre of the Type C octagon is surrounded by 336 yods. Weighted with the number 10 (Decad), the 672 yods surrounding the centres of the two Type C octagons that are part of the inner Tree of Life generates the number 6720. This is the number of edges of the 4_{21} polytope. When they are Type B, the pair of octagons have 240 yods surrounding their centres. This is the number of vertices of the 4_{21} polytope. Can it be coincidence that *both* the number of vertices of the 4_{21} polytope and its number of edges are embodied in *consecutive* types of octagons? Of course not! Instead, it demonstrates how the Godnames of the Sephiroth constitute powerful, mathematical archetypes that determine through sacred geometry and number the very nature of the divine paradigm.

Figure 9

אל חאי = 49



49 yods

336 yods surround centre

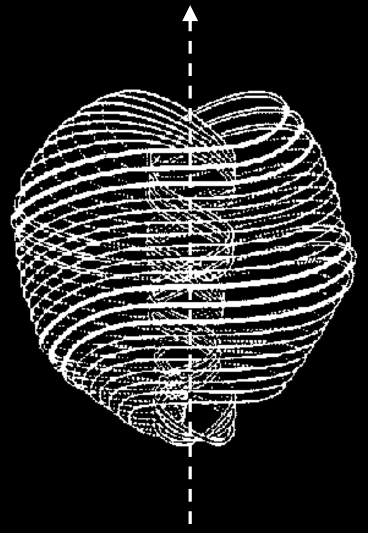
336 yods surround centre

The number value **49** of EL CHAI, the Godname of Yesod, is the number of yods in an octagon with tetractyses as sectors. 336 yods surround the centre of a Type C octagon. Weighted with the Decad, the 672 yods surrounding the centres of the two Type C octagons generate the number (6720) of edges of the 4_{21} polytope.

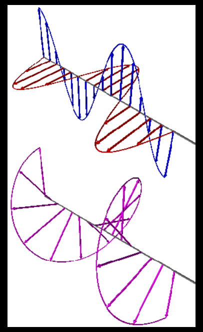
Commentary on Fig. 10

The 4_{21} polytope has 240 vertices whose position vectors in 8-dimensional space denote the 240 roots of the exceptional Lie group E_8. It has 6720 edges — 3360 edges in each half of the polytope. The UPA described by Annie Besant and C.W. Leadbeater with the yogic siddhi of anima consists of 10 helical, closed curves, or “whorls,” each with 1680 circular turns. A whorl makes five revolutions about the axis of spin of the UPA, each revolution containing 336 turns. The 10 helical whorls contain 3360 turns in each revolution. Each turn is a circularly polarised oscillation that is a supposition of two plane waves vibrating in perpendicular planes 90° out of phase with each other. This means that the 10 whorls make (3360×2=6720) plane wave oscillations in every revolution around the spin axis (672 oscillations in each whorl). Unless we are willing to accept it as a miracle of coincidence, it is clear that, because the number of edges of the 4_{21} polytope is equal to the number of plane wave oscillations in one revolution of all 10 whorls of the UPA, it must be a geometrical expression of the latter as a subquark state of the $E_8 \times E_8$ heterotic superstring, the 240 vertices expressing the 240 E_8 gauge charges that are spread along its whorls. As shown in previous pages, the fact that sacred geometries embody the number 672 is not a coincidence. Instead, it expresses the fact that each of the **50** revolutions of the whorls is a *whole* in itself.

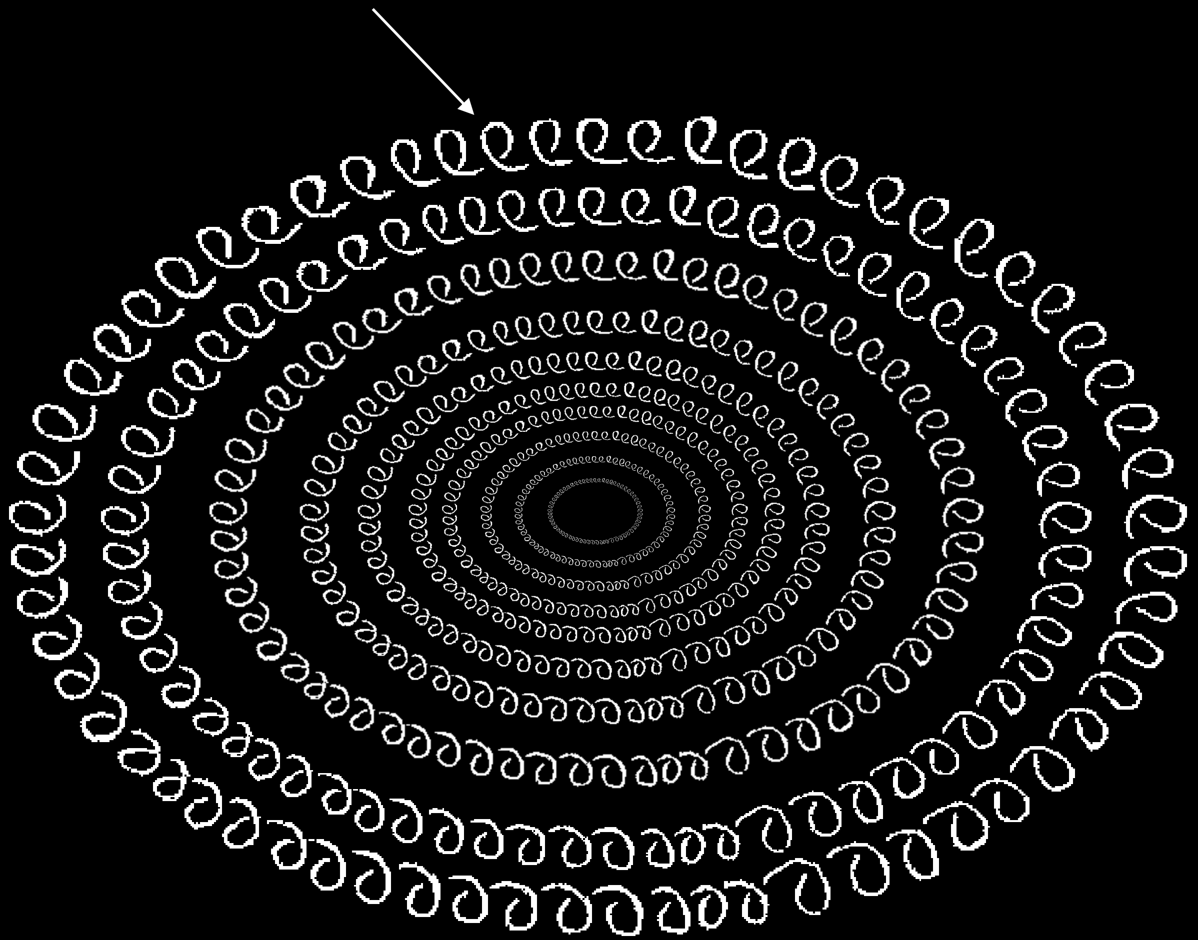
Figure 10

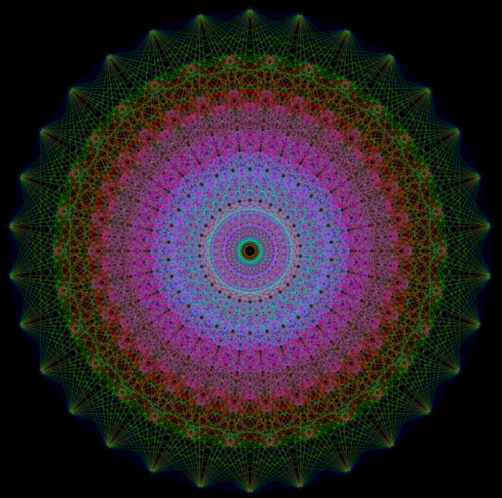


The 10 whorls of the UPA revolve five times around its axis of spin



The 3360 circularly polarised waves in one revolution of the 10 whorls of the UPA have 6720 plane wave components





The 4_{21} polytope, whose 240 vertices define the 240 roots of E_8 governing $E_8 \times E_8$ heterotic superstring forces, has 6720 edges