

ARTICLE 25

The 33 Vertex Sheets of the Disdyakis Triacontahedron Signify the 33 Tree Levels of Ten Overlapping Trees of Life

by

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Abstract

This article elucidates the scientific meaning of the patterns of layers of the three types of vertices of the disdyakis triacontahedron. This Catalan solid was identified in Articles 22–24 as the 3-dimensional (polyhedral) version of the 2-dimensional (polygonal), inner form of the Tree of Life. Its 62 vertices are the counterpart of the 62 Sephirothic emanations of 10 overlapping Trees of Life that are shared with their inner forms. Its vertices are arranged in either 7, 11 & 15 sheets perpendicular to axes that join, respectively, pairs of opposite A, B & C vertices. The 33 sheets correspond to the 33 tree levels (rungs of the biblical 'Jacob's ladder') of 10 overlapping Trees of Life. Within the inner form of a single Tree of Life, they correspond to the 33 corners outside the shared edge of the set of 7 enfolded polygons that, being intrinsic to that set, represent geometrical degrees of freedom that are unique to any Tree of Life system. The 32 B & C vertices of the golden rhombic faces of the triacontahedron underlying the disdyakis triacontahedron span 26 sheets prescribed by the Divine Name YHVH. The number value 15 of YH is the number of sheets orthogonal to a C-C axis and the number value 11 of VH is the number of sheets orthogonal to a B-B axis. The 7 sheets orthogonal to an A-A axis correspond to the 7 tree levels spanning the lowest of the 10 Trees of Life; they determine the 84 circularly polarised oscillations made by a whorl of the $E_8 \times E_8$ ' heterotic superstring during one-quarter of a revolution about its spin axis. The 11 B-B sheets correspond to the next 11 tree levels; they determine the 168 oscillations in a half-revolution of a whorl. The 15 C-C sheets correspond to the final 15 tree levels in 10 overlapping Trees of Life; they determine the 240 gauge charges associated with the non-zero roots of the superstring gauge symmetry group E_8 that are spread along the length of the heterotic superstring. Assigning 4, the Pythagorean Tetrad, to the 62 vertices of the disdyakis triacontahedron generates the dimension 248 of E_8 . Assigning 4 to the 420 yods in the 120 tetractyses needed to construct the sacred geometry of the disdyakis triacontahedron from two diametrically opposite A vertices generates the number 1680. This is the number of circularly polarised oscillations in each whorl of the heterotic superstring, as well as the number of hexagonal yods forming the seven types of polyhedra leading to the disdyakis triacontahedron. This polyhedron therefore embodies the dynamics and oscillatory form of the $E_8 \times E_8$ ' heterotic superstring as the manifestation of the Tree of Life blueprint in the subatomic world.

1. Tree levels as space-time dimensions

Just as the DNA molecule in the nucleus of a cell encodes the properties of the biological organism made up of such cells, so the inner form of the Tree of Life (Fig. 1) encodes how its outer form replicates to represent the *whole* cosmos — both physical and superphysical. By exploiting the isomorphism between a regular, n-sided polygon divided into tetractyses and n overlapping Tree of Life up to Chesed of the nth tree, the author has proved rigorously that the two sets of seven separate polygons contain a unique subset of 12 polygons that is isomorphic to a whole number of overlapping

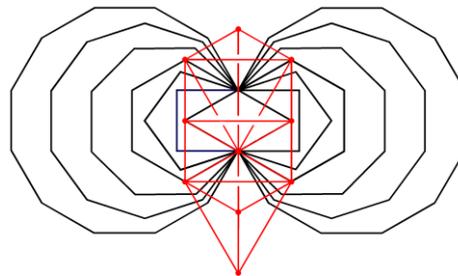


Figure 1. The outer (red) and inner (black) Tree of Life.

Trees of Life. One set of seven polygons is isomorphic to 49^1 such trees and the remaining five polygons (the pentagon, hexagon, octagon, decagon and dodecagon of the second set of polygons) are isomorphic to 42 trees. Encoded, therefore, in the regular polygons making up the inner Tree of Life are 91 overlapping Tree of Life.

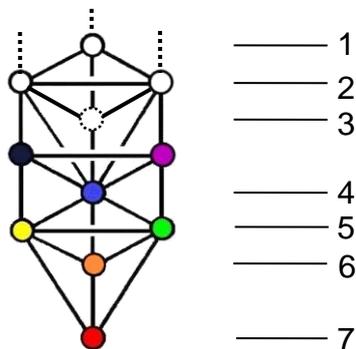


Figure 2. The seven tree levels.

49 trees represent the **49** subplanes of the seven planes of consciousness — one of the teachings of Theosophy that can now be mathematically proved. These seven planes represent the outer evolutionary journey of the soul from one incarnation to another towards absorption in the inner Divine Life. They constitute but the lowest physical plane of seven cosmic planes of consciousness, the six higher planes of which are similarly divided into seven subplanes, making a total of 42 subplanes. These are represented by the 42 trees encoded in the five polygons belonging to the second set of seven polygons. Accordingly, the two halves of the inner Tree of Life signify the physical and

superphysical domains of the spiritual cosmos, which the author has called the 'Cosmic Tree of Life' (CTOL). The boundary between these two domains is prescribed by EL CHAI, the Godname of Yesod, which has number value **49** (the other Godnames also prescribe it in ways that need not be described here).

The seven lowest trees in CTOL represent the seven levels of biological consciousness. In their normal physical activity, humans are focussed in the third tree but are evolving as well into 'etheric' realms of consciousness that are mapped by the next four trees. Defining the 'n-tree' as the lowest n trees in CTOL, physical consciousness can, potentially, extend over the 7-tree. When trees overlap, all the Sephirothic emanations (called 'Sephirothic levels,' or SLs) of one tree are SLs of the next higher and lower tree except Chesed and Geburah. These two types of SLs play a pre-formative role in the manifestation of the next lower tree as a level of consciousness. They represent gaps, or critical jumps, in the spectrum of awareness as it shifts from one modality to the next represented by the SLs of the next higher tree.

Each level of consciousness is a band of types of awareness characterised by the seven fundamental Sephiroth of Construction. The lowest four relate to what Carl Jung called the four 'psychic functions' of sensing, feeling, thinking and intuiting. These Sephirothic variations define seven 'tree levels' within each tree/level of consciousness (Fig. 2). The fourth has special evolutionary significance because it expresses the quality of Tiphareth. As the central Sephirah of the Tree of Life, it is the interface between the ego-centred and transpersonal, ego-less levels of being. The number of tree levels in the n-tree $\equiv T(n) = 3n + 4$. Hence, $T(7) = 25$. They denote the 25 spatial

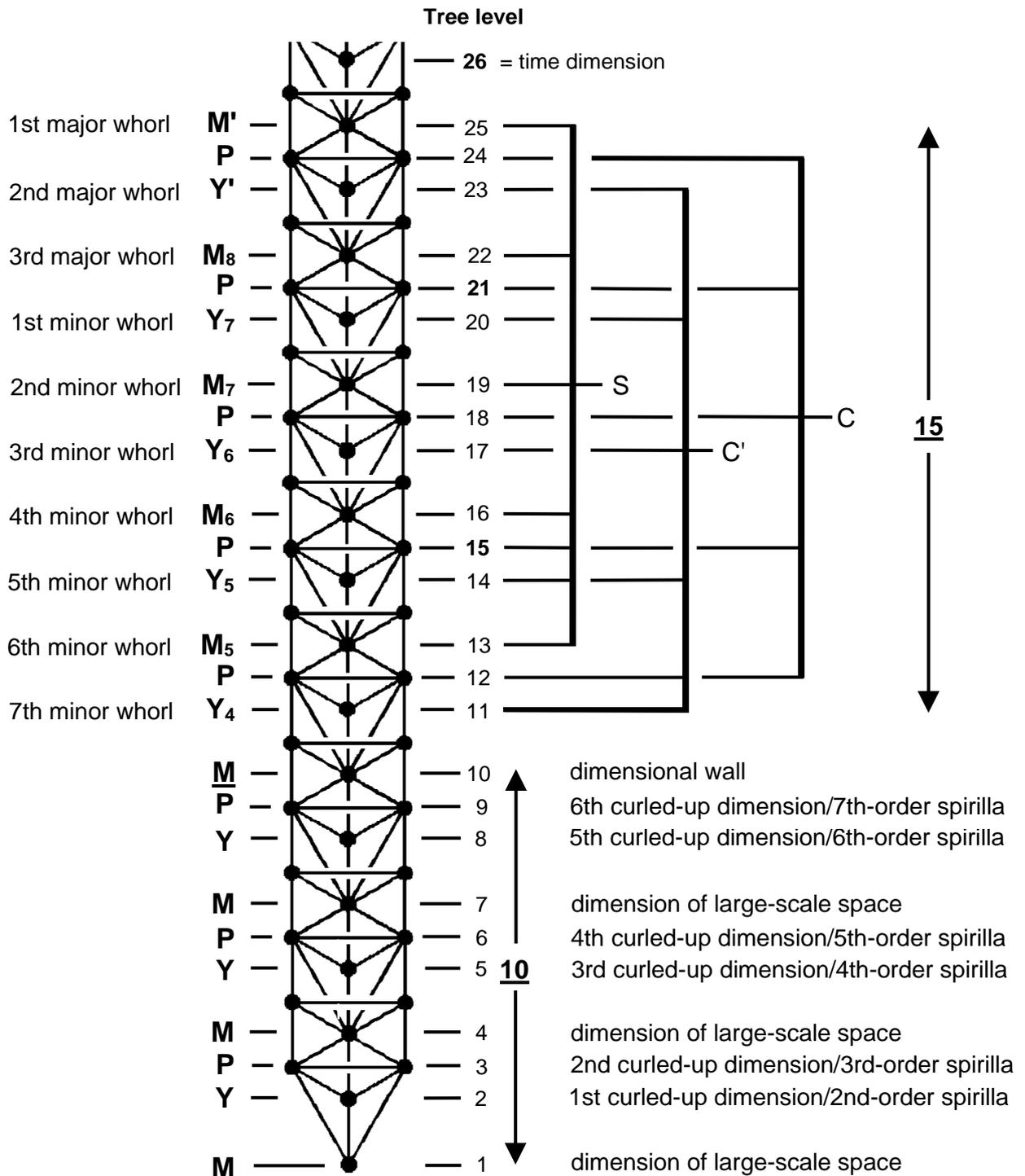
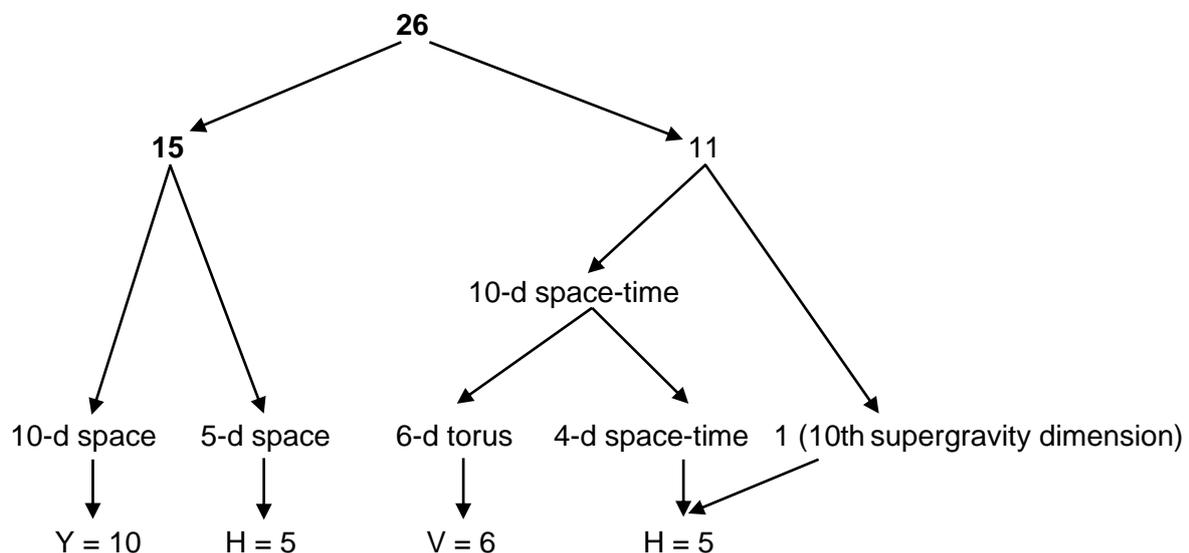


Figure 3. The Tree of Life mapping of 26-dimensional space-time.

dimensions of the **26**-dimensional space-time predicted by quantum mechanics for spinless strings. The **26**th tree level (the lowest one above the 7-tree) maps the time dimension. It is marked by Yesod of the ninth tree, the **50**th SL. This shows how ELOHIM, Godname of Binah with number value **50**, prescribes the **26** dimensions of space-time, which are prescribed directly by YAHWEH, Godname of Chokmah, because it has number value **26**. YAH, the older version of Yahweh, has number **15**. It prescribes the 25 dimensions of space because the top of the seventh tree marked by the 25th tree level is the 47th SL, where 47 is the **15**th prime number. The Godname EL of Chesed also prescribes these 25 dimensions because its number value **31** is the number of stages of vertical descent of what Kabbalists call the 'Lightning Flash' from the apex of the seventh tree to the nadir of CTOL, namely, Malkuth of the first tree.

There are three types of tree levels: Malkuths (M) of trees, Yesods (Y) and Hod-Netzach Paths (P) (apart from the lowest tree, the lattermost are Binah-Chokmah Paths of the next lower tree). Article 2 explained the role of these tree levels in creating the ten whorls of the UPA described by Annie Besant and C.W. Leadbeater in their book *Occult Chemistry*. The author has identified the UPA as a heterotic $E_8 \times E_8$ superstring,² each of its ten whorls resulting from the curling into a circle of a dimension represented by a tree level (Fig. 3). Five M and five Y tree levels belonging to the uppermost four trees define a 10-dimensional space $S \times C'$ whose curling up generates the 10 whorls. Five P tree levels define a 5-dimensional space C whose symmetries generate the E_8 gauge symmetry group. The remaining 10-dimensional space is that of supergravity theories, one of the dimensions being a finite segment between the two 10-dimensional space-times of ordinary and shadow matter superstrings.³

The dimensionalities of the various spaces into which the author's theory predicts **26**-dimensional space-time bifurcates are the letter values of YHVH (YAHWEH), the



later version of the Godname of Chokmah given by God to the Jewish people, according to their religious beliefs. The value 10 of Y (yod) denotes the 10 dimensions of $S \times C'$ whose curling up creates the ten whorls of the UPA, the value 5 of H (he) denotes the 5-dimensional space C whose geometrical symmetries generate E_8 , the value 6 of V (vav) denotes the curled-up, 6-dimensional space of superstrings and the value 5 of the second H denotes the Einsteinian, 4-dimensional space-time and the dimensional segment separating the two superstring space-times. According to Leadbeater's description of the UPA, each whorl is a closed helix with 1680 turns

("1st-order spirillae"). Each turn is a helix with seven circular turns ("2nd-order spirillae"); each of the latter is a smaller helix with seven turns ("3rd-order spirillae") and so on. The six higher orders of spirillae described by Leadbeater represent the winding of strings around successively smaller, compact, 1-dimensional spaces, i.e., the 6-dimensional, compactified space of superstrings is a 6-d torus. Figure 3 shows that tree levels of type P and Y define these curled-up dimensions.

The superstrings of ordinary and shadow matter are confined to two space-time sheets separated by a gap. This tenth dimension acts as a wall that prevents either kind of superstring entering the other's universe. It is represented by the tenth tree level (M in Fig. 3). This is Malkuth of the fourth highest tree in the 7-tree mapping the physical plane, that is, **26**-dimensional space-time). This **15:11** division between the dimensionalities of supergravity space-time and the higher, **15**-dimensional space that generates both the ten string-like whorls of the UPA/heterotic superstring and its gauge symmetry group E_8 has its counterpart in the combinatorial properties of ten objects labelled A, B, C.... J when arranged in the pattern of a tetractys:

n	Combinations	Number of combinations = $2^n - 1$
1	A	1
2	B C	3
3	D E F	7
4	G H I J	15
		TOTAL = <u>26</u>

The four objects G, H, I and J have ($2^4-1=15$) combinations. The three objects D, E and F have ($2^3-1=7$) combinations, the two objects B and C have ($2^2-1=3$) combinations and the single object A has one combination. The six objects A–F therefore form 11 combinations. When arranged in a tetractys, all ten objects have **26** combinations when only objects from the same row are considered.

This has a simple geometrical realisation. Letting the objects be points in space, the four rows of the tetractys of ten points signify the sequence of point, line, triangle and tetrahedron. Combinations of two points denote the line joining them, combinations of three points signify the triangles with these points as their corners and the combination of four points signifies the tetrahedron with these points as its vertices. There are **26** vertices, lines, triangles and tetrahedra in the 4-stage sequence of generation from the point of the simplest Platonic solid — the tetrahedron. This illustrates in a very simple way the mathematical archetype embodied in the Divine Name YAHWEH.

n objects have n! permutations. The permutations of the objects in a tetractys are:

n	Permutations	Number of permutations = n!
1	A	1
2	B C	2
3	D E F	6
4	G H I J	24
		TOTAL = <u>33</u>

There are **26** combinations and 33 permutations of the objects arranged in the four rows of a tetractys when only objects in the same row are grouped together.

The tetractys and the Tree of Life are different representations of the 10-fold nature of holistic systems designed according to the universal blueprint. Each Sephirah of the Tree of Life has in turn a 10-fold differentiation, so that ten overlapping Trees of Life is the next higher differentiation of a single Tree of Life. In the same way, the 10-tree is

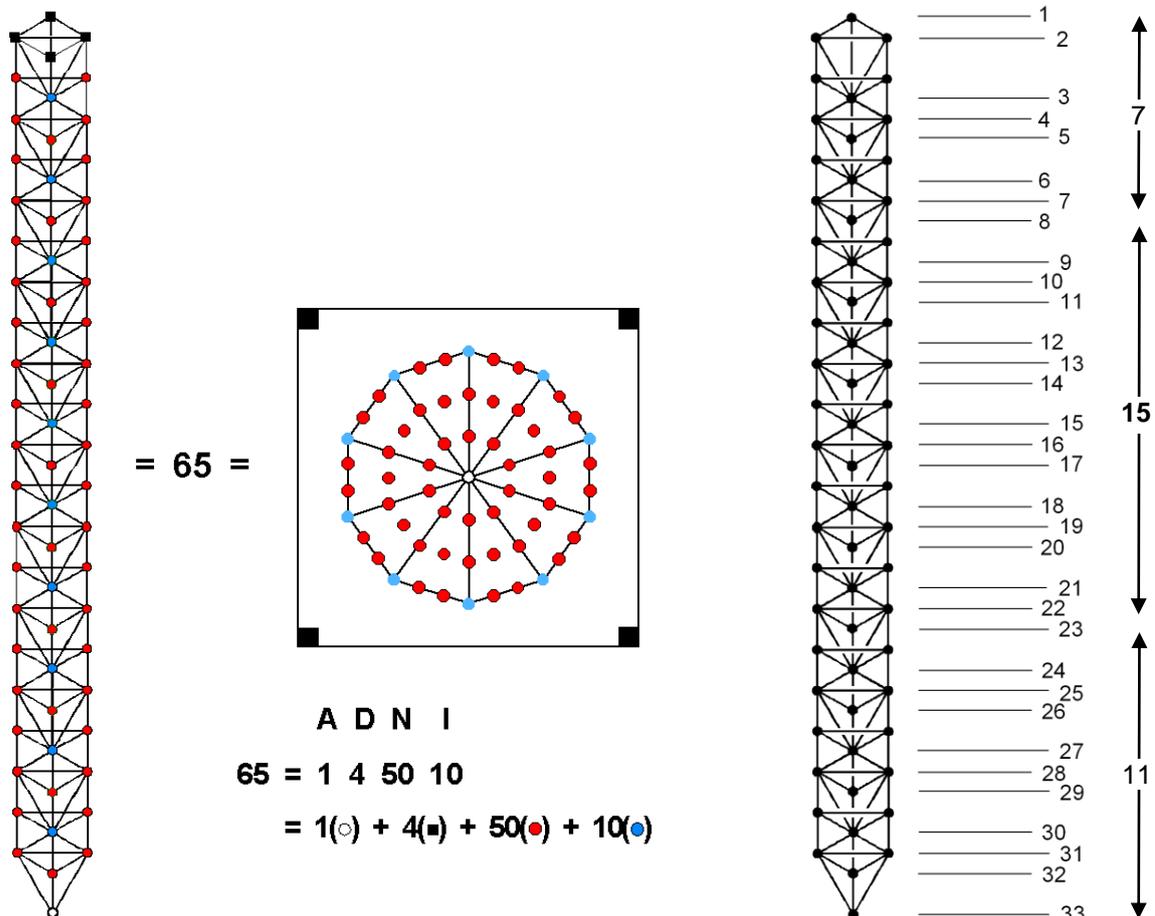


Figure 4. Equivalence of the 10-tree prescribed by ADONAI and its tetractys representation. Ten overlapping Trees of Life have 33 tree levels.

the next higher differentiation of the 1-tree. It has **65** SLs (Fig. 4). **65** is the number value of the Godname ADONAI assigned to Malkuth. In fact, its very letter values signify the numbers of various types of SLs in the 10-tree. This is borne out by the representation of the 10-tree as a decagon divided into 10 tetractyses and surrounded by a square, the letter values denoting the yods that are counterparts of the SLs. The **64** SLs of the ten overlapping trees span 33 tree levels. We saw earlier that the lowest **50** SLs span **26** tree levels signifying the **26** dimensions of space-time predicted by quantum mechanics for spin-0 strings. The 25 spatial dimensions/tree levels emanates from the **50th** SL, which, as Daath, represents in Kabbalah the 'Abyss' between the Supernal Triad represented by the three uppermost trees of the ten overlapping trees and the seven Sephiroth of Construction represented by the seven lowest trees. Hence, the division of tree levels in ten overlapping Trees of Life:

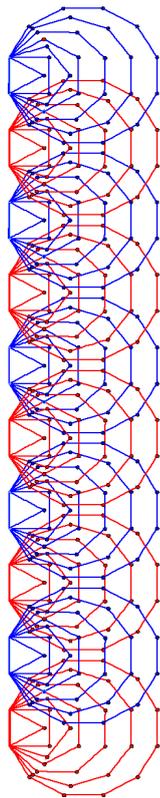
$$33 = 7 + 26$$

expresses in Kabbalistic terms the distinction between the seven Sephiroth of Construction and the triple Godhead of Kether, Chokmah and Binah.

The number of corners of the $7n$ regular polygons enfolded in n overlapping Trees of Life is

$$C(n) = 35n + 1,$$

where “1” denotes the uppermost corner of the hexagon enfolded in the tenth tree (the hexagon is the only one of the seven regular polygons that shares its corners with polygons enfolded in adjacent trees. It results in its being picked out by the above formula). Therefore, $C(10) = 351 = 1 + 2 + 3 + \dots + 26$. This shows how YAHWEH with number value **26** prescribes the inner form of ten overlapping Trees of Life. **351** is the



**33 (●) or (●) corners
per set of 7 polygons**

Figure 5. The total number of corners of the 70 polygons enfolded in 10 trees is **351**, where **351** = 1 + 2 + 3 + ... + **26**. This is how the Divine Name YAHWEH with number value **26** prescribes the inner form of 10 Trees of Life. There are 34 corners in each set of seven polygons outside their root edge. As the uppermost corner of the hexagon coincides with the lowest corner of the hexagon enfolded in the next higher tree, there are 33 corners per set of polygons. These independent, geometrical degrees of freedom correspond to the 33 tree levels of 10 overlapping Trees of Life.

number value of *Ashim* (“Souls of Fire”), the Order of Angels assigned to Malkuth. The number of corners of the $7n$ polygons outside their n root edges $\equiv C'(n) - 2n = 33n + 1$. $C'(n+1) - C'(n) = 33$. In other words, there are 33 corners per set of polygons outside their root edge (Fig. 5). *The emanation of successive Trees of Life generates 33 new geometrical degrees of freedom associated with their inner form.* The significance of this property for the disdyakis triacontahedron will be discussed in the next section.

The 10-tree has 34 tree levels, whilst ten overlapping Trees of Life have 33 tree levels. Their counterparts in the inner Tree of Life are its 34 corners outside the root edges, 33 per set of polygons. Any system or structure that conforms to the blueprint of the Tree of Life possesses 33 independent degrees of freedom. The counterpart of this in the tetractys is the 33 permutations of objects in its four rows.

The seven polygons enfolded on one side of the central Pillar of Equilibrium have corners shared with Chokmah, Daath, Chesed and Netzach. The other seven polygons

have corners shared with Binah, Tiphareth, Geburah and Hod. The top corner of the hexagon is the bottom corner of the hexagon enfolded in the next higher tree. This means that there are (3+1) corners associated with each set of polygons, “1” denoting the shared, top corner of the hexagon. Enfolded in n overlapping Trees of Life are $7n$ polygons on either side of the central pillar with $(3n+1)$ corners that coincide with SLs. The 10-tree has 70 polygons on each side with **31** corners shared with it. Hence, there are **62** SLs of the 10-tree that are shared with its inner form. The **62** vertices of the disdyakis triacontahedron correspond to these SLs (Fig. 6), the **31** independent vertices and their **31** inverted images corresponding to the **31** SLs that coincide with corners of each set of polygons. **31** is the number value of EL (“God”), the Godname of Chesed, and **62** is the number value of *Tzadkiel* (“Benevolence of God”), the Archangel of Chesed — the *same* Sephirah.

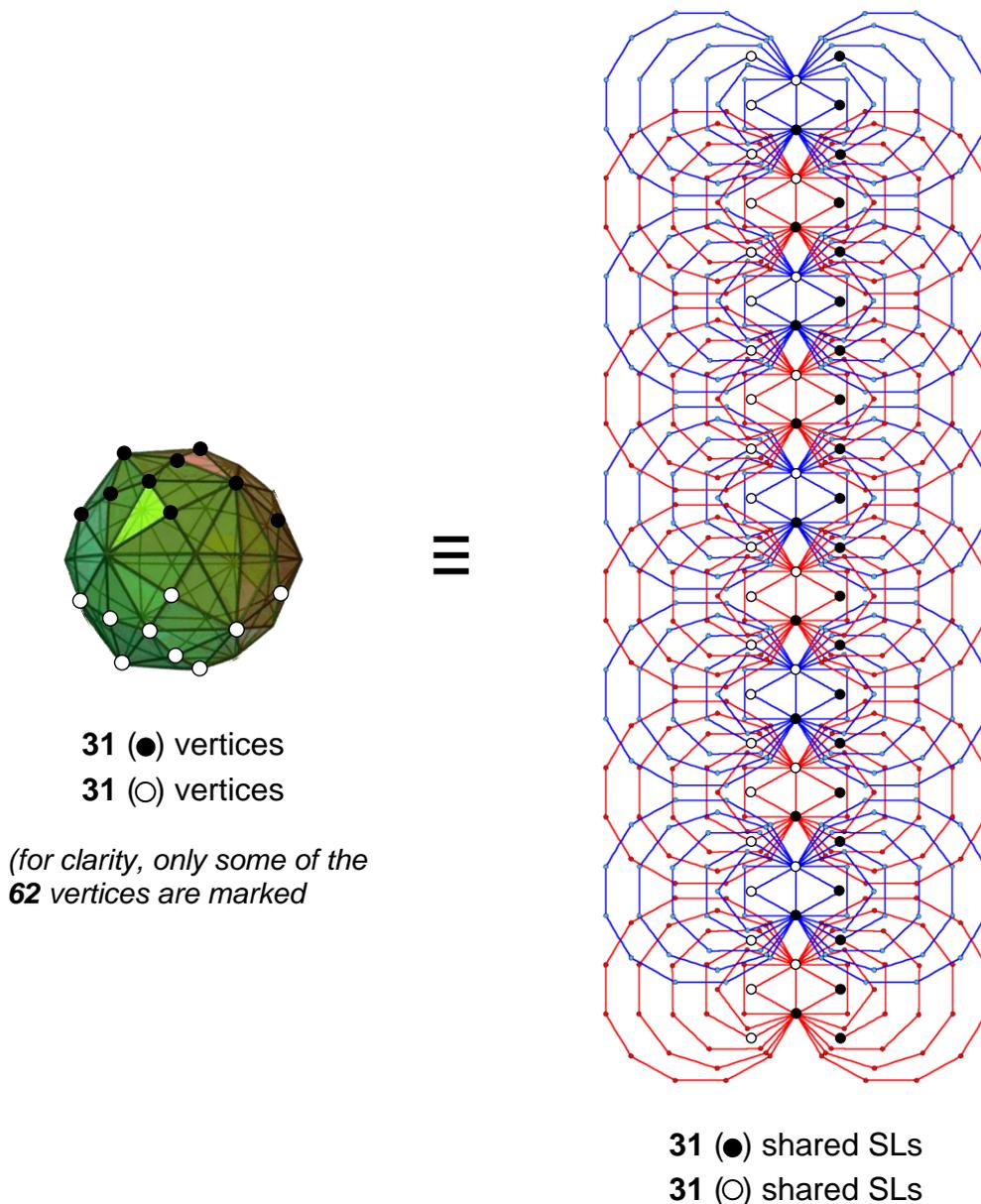


Figure 6. The **31** vertices of the disdyakis triacontahedron and their inversions correspond to the **31** SLs of the 10-tree and their mirror images that are shared with its inner, polygonal form. The number of unshared corners is **620**, which is the number value of Kether (“Crown”), the first Sephirah of the Tree of Life.

2. Vertex planes in the disdyakis triacontahedron

The following evidence was presented in Article 22 that the disdyakis triacontahedron is the 3-dimensional realisation of the 2-dimensional (polygonal) form of the inner Tree of Life:

1. the 120 shape-forming yods on the boundaries of the seven enfolded polygons symbolise the 120 faces of this polyhedron;
2. the disdyakis triacontahedron completes the 10-step sequence of generation of polyhedra from the mathematical point:
 1. point
 2. line
 3. triangle
 4. tetrahedron
 5. octahedron
 6. cube
 7. icosahedron
 8. dodecahedron
 9. rhombic triacontahedron
 10. disdyakis triacontahedron;
3. the 480 hexagonal yods in the 120 faces of the disdyakis triacontahedron correspond to the 480 hexagonal yods in the two sets of seven regular polygons constituting the inner Tree of Life;
4. the **62** vertices of the disdyakis triacontahedron correspond to the **62** SLs of 10 overlapping Trees of Life that coincide with corners of their **140** enfolded polygons and to the **62** corners of the 14 polygons enfolded in one Tree of Life that do not coincide with its Sephiroth;
5. the seven polyhedra in the above sequence are made up of 1680 hexagonal yods — the number of yods below the top of the 10-tree when its triangles are divided into three tetractyses. This is the structural parameter of the $E_8 \times E_8'$ heterotic superstring, as established in many previous articles and in the author's books;
6. the hexagonal yod population of the 28 Platonic and Catalan solids in the disdyakis triacontahedron is 3360. This is the yod population of the seven enfolded polygons when constructed from 2nd-order tetractyses.⁴

Further evidence in support of this conclusion is provided by the patterns of planes of vertices of the disdyakis triacontahedron. Its **62** vertices are made up of the 32 vertices of a rhombic triacontahedron, which form 30 Golden Rhombi, and 30 raised centres of the latter. The long diagonals of each Golden Rhombus are the edges of an

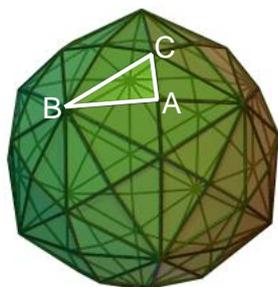


Figure 7. The three types of vertices of the disdyakis triacontahedron.

icosahedron and its short diagonals are the edges of a dodecahedron. Labelling the vertices of the 120 triangular faces A, B & C (Fig. 7), there are 30 A vertices, 12 B vertices (corners of the icosahedron) and 20 C vertices (corners of the dodecahedron).

Remarkably, this pattern of types of vertices appears also in the inner Tree of Life in the 62 corners of the two sets of enfolded polygons that are unshared with Sephiroth of its outer form (Fig. 8). The two dodecagons have 20 corners (red yods) outside the root

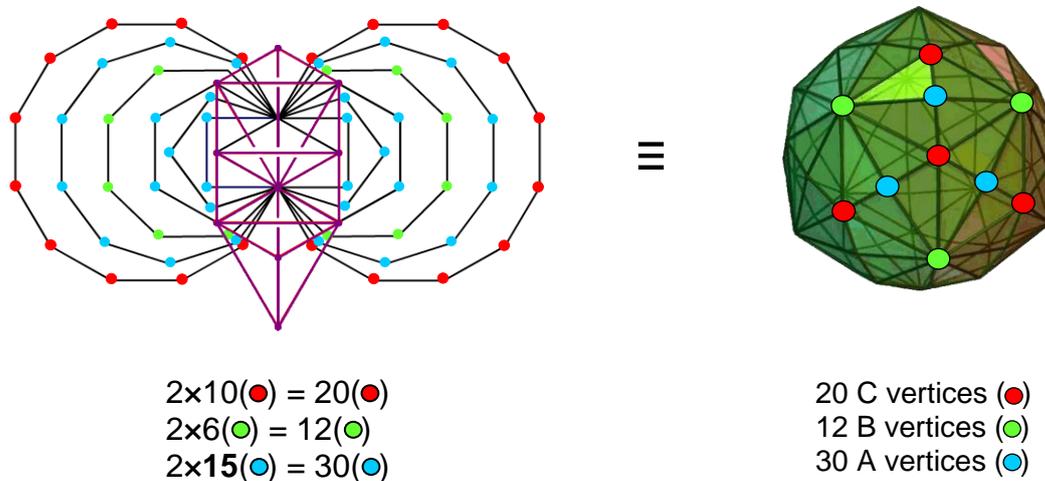


Figure 8. Eight of the 70 corners of the inner Tree of Life (shown as black lines) are shared with the outer Tree of Life (shown as violet lines), leaving **62** unshared corners. These correspond to the **62** vertices of the disdyakis triacontahedron which is its 3-dimensional realisation. The **31** unshared corners on each side of the shared root edge correspond to the **31** vertices and their inversions through the centre of the disdyakis triacontahedron. The 20 unshared corners of the two dodecagons correspond to the 20 C vertices, the 12 unshared corners of the two octagons correspond to the 12 B vertices and the 30 unshared corners of the square, pentagon, hexagon and decagon correspond to the 30 A vertices.

edge. The pair of octagons has 12 corners outside the root edge and the pairs of squares, pentagons, hexagons and decagons have 30 corners outside the root edge. We find the following correspondence:

- 30 corners of squares, pentagons, hexagons and decagons → 30 A vertices
- 12 corners of octagons → 12 B vertices
- 20 corners of dodecagons → 20 C vertices

For the sake of clarity, only three representative A, B and C vertices are shown in the disdyakis triacontahedron in Fig. 7. It may be argued that the numbers of unshared corners in pairs of polygons:

square	pentagon	hexagon	octagon	decagon	dodecagon
4	6	4	12	16	20

allow other combinations of polygons to generate the numbers 30, 12 and 20, e.g., the pair of squares, pentagons and dodecagons have 30 corners. This is not true of the number 12, which can be associated only with the pair of octagons because it is not the sum of any of the above numbers. The dodecagons are the simplest choice for the number 20 because other choices require combinations of two or three polygons. Any argument over uniqueness, however, misses the point being demonstrated here, namely, that correspondence can be established not only between the numbers of vertices of the disdyakis triacontahedron and the corners of the inner Tree of Life but also between the numbers of each type of vertex and numbers of corners of different

Vertex	Type	X	Y	Z
1	A	0	0	$2\phi^2$
2	B	ϕ^2	0	ϕ^3
3	A	ϕ	ϕ^2	ϕ^3
4	C	0	ϕ	ϕ^3
5	A	$-\phi$	ϕ^2	ϕ^3
6	B	$-\phi^2$	0	ϕ^3
7	A	$-\phi$	$-\phi^2$	ϕ^3
8	C	0	$-\phi$	ϕ^3
9	A	ϕ	$-\phi^2$	ϕ^3
10	A	ϕ^3	ϕ	ϕ^2
11	C	ϕ^2	ϕ^2	ϕ^2
12	B	0	ϕ^3	ϕ^2
13	C	$-\phi^2$	ϕ^2	ϕ^2
14	A	$-\phi^3$	ϕ	ϕ^2
15	A	$-\phi^3$	$-\phi$	ϕ^2
16	C	$-\phi^2$	$-\phi^2$	ϕ^2
17	B	0	$-\phi^3$	ϕ^2
18	C	ϕ^2	$-\phi^2$	ϕ^2
19	A	ϕ^3	$-\phi$	ϕ^2
20	C	ϕ^3	0	ϕ
21	A	ϕ^2	ϕ^3	ϕ
22	A	$-\phi^2$	ϕ^3	ϕ
23	C	$-\phi^3$	0	ϕ
24	A	$-\phi^2$	$-\phi^3$	ϕ
25	A	ϕ^2	$-\phi^3$	ϕ
26	A	$2\phi^2$	0	0
27	B	ϕ^3	ϕ^2	0
28	C	ϕ	ϕ^3	0
29	A	0	$2\phi^2$	0
30	C	$-\phi$	ϕ^3	0
31	B	$-\phi^3$	ϕ^2	0
32	B	ϕ^3	$-\phi^2$	0
33	C	ϕ	$-\phi^3$	0
34	A	0	$-2\phi^2$	0
35	C	$-\phi$	$-\phi^3$	0
36	B	$-\phi^3$	$-\phi^2$	0
37	A	$-2\phi^2$	0	0
38	A	$-\phi^2$	ϕ^3	$-\phi$
39	A	ϕ^2	ϕ^3	$-\phi$
40	C	ϕ^3	0	$-\phi$
41	A	ϕ^2	$-\phi^3$	$-\phi$
42	A	$-\phi^2$	$-\phi^3$	$-\phi$
43	C	$-\phi^3$	0	$-\phi$
44	A	$-\phi^3$	ϕ	$-\phi^2$
45	C	$-\phi^2$	ϕ^2	$-\phi^2$
46	B	0	ϕ^3	$-\phi^2$
47	C	ϕ^2	ϕ^2	$-\phi^2$
48	A	ϕ^3	ϕ	$-\phi^2$
49	A	ϕ^3	$-\phi$	$-\phi^2$
50	C	ϕ^2	$-\phi^2$	$-\phi^2$
51	B	0	$-\phi^3$	$-\phi^2$
52	C	$-\phi^2$	$-\phi^2$	$-\phi^2$
53	A	$-\phi^3$	$-\phi$	$-\phi^2$
54	A	$-\phi$	ϕ^2	$-\phi^3$
55	C	0	ϕ	$-\phi^3$
56	A	ϕ	ϕ^2	$-\phi^3$
57	B	ϕ^2	0	$-\phi^3$
58	A	ϕ	$-\phi^2$	$-\phi^3$
59	C	0	$-\phi$	$-\phi^3$
60	A	$-\phi$	$-\phi^2$	$-\phi^3$
61	B	$-\phi^2$	0	$-\phi^3$
62	A	0	0	$-2\phi^2$

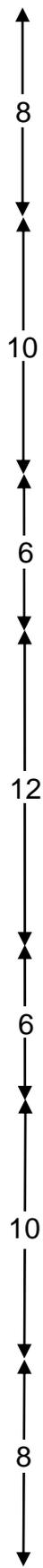


Table 1

The coordinates of the **62** vertices of the disdyakis triacontahedron arranged according to their height (z coordinate). Vertices in a section with the same colour have the same height. The number of vertices within each such plane is indicated within an arrow. Vertex 1 is the apex of the disdyakis triacontahedron and vertex **62** is its nadir. The 60 vertices between them are arranged in seven sheets. The pattern of **31** vertices below the thick line is the mirror image of the pattern of **31** vertices above it.

polygons. Such *precise* correspondence cannot be dismissed as due to chance. It is further evidence that the disdyakis triacontahedron is the 3-dimensional form of the inner Tree of Life.

Table 1 lists the Cartesian coordinates of the **62** vertices. Their coordinates are the first three powers of ϕ , the Golden Ratio. They are arranged in nine vertical layers, the

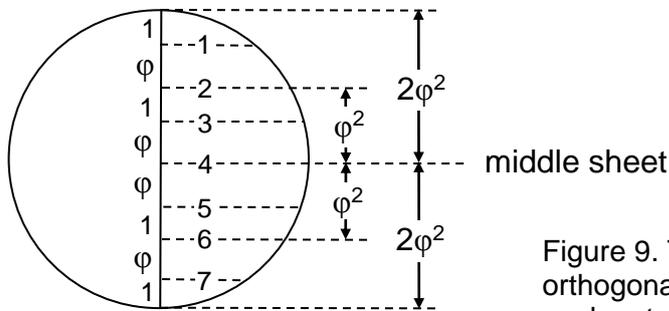


Figure 9. The seven sheets of vertices orthogonal to an A-A axis. Their separations in each set of four sheets alternate as 1, ϕ , 1, ϕ .

vertical axis being defined as the line joining the top and bottom A vertices indicated by the two white rows. The 60 vertices between them are distributed in seven horizontal

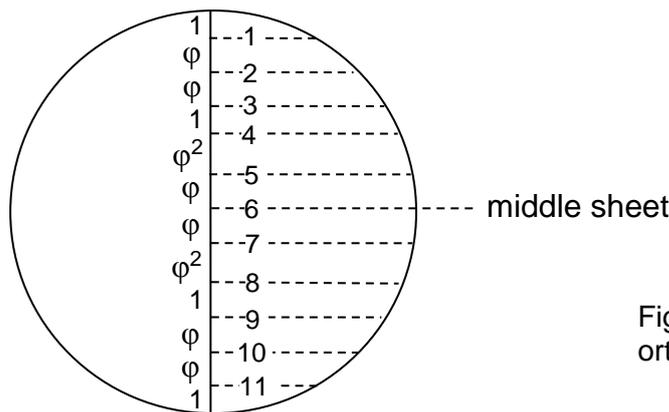


Figure 10. The 11 sheets of vertices orthogonal to a B-B axis.

planes coloured according to the seven prismatic colours. The **62** sets of three coordinates are really **31** sets, as the other **31** vertices are the inversions of those in the first set, the coordinate values of one vertex being the negatives of its inverted counterpart. This is a clear example of the prescriptive power of the mathematical

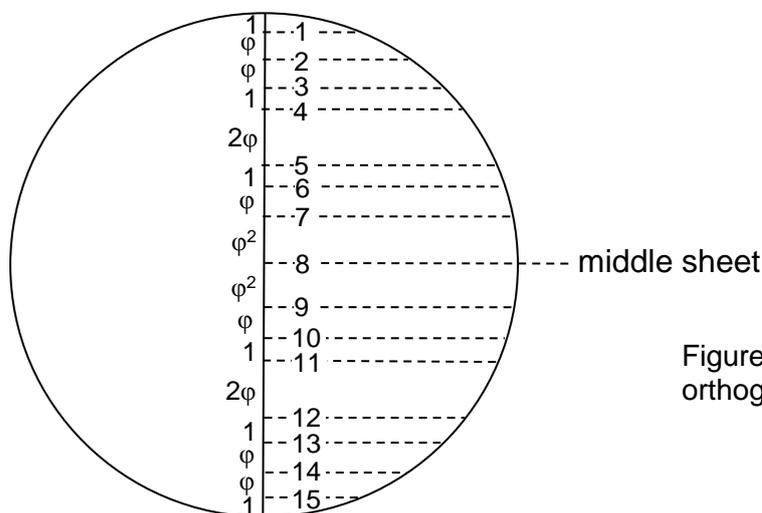


Figure 11. The **15** sheets of vertices orthogonal to a C-C axis.

archetype embodied in the Godname EL (“God”) associated with Chesed, the first Sephirah of Construction. Its letter vales E = 1 and L = 30 denote, respectively, the topmost vertex and the 30 vertices below it that have mirror image counterparts.

Fig. 9 indicates that the 30 A vertices are arranged in nine layers. The seven sheets between the outermost vertices are separated by distances that alternate as 1, φ, 1, φ, this pattern being reversed for the last four sheets.

Fig. 10 indicates that the 12 B vertices (the corners of an icosahedron) are arranged in 11 sheets between the opposite B vertices defining the axis at right angles to them.

Fig. 11 shows that the 20 C vertices (corners of a dodecahedron) belong to 15 sheets between the opposite C vertices defining the axis. The disdyakis triacontahedron has therefore (7+11+15=33) sheets.

3. Tree levels signify sheets of vertices

Comparing the sheet patterns of the three types of vertices with the tree levels of 10 overlapping Trees of Life shown in Fig. 4, we see that there are as many sheets as there are tree levels. Moreover, there are as many sheets (7) perpendicular to an A-A axis as there are tree levels in 10 overlapping Trees of Life above the 26th tree level marking the 50th SL, there being 26 sheets perpendicular to a B-B and a C-C axis. This is the number value of YAHWEH. The distinction between the 11 B-B sheets and the 15 C-C sheets is expressed in the letter values of YAHWEH:

$$\begin{array}{cc|cc} Y & H & V & H \\ 10 & 5 & 6 & 5 \\ \hline & 15 & & 11 \end{array}$$

It prescribes the distribution of vertices perpendicular to B-B and C-C axes, whilst YAH, the older version of this divine name with number value 15, prescribes the distribution of vertices perpendicular to a C-C axis.

The fact that the same three numbers 7, 15 and 11 appear in the contexts of 10 overlapping Trees of Life and the disdyakis triacontahedron is additional confirmation that the latter is equivalent to the inner form of the Tree of Life, its 62 corners unshared with its outer form being the counterpart of the 62 SLs of 10 trees that are unshared

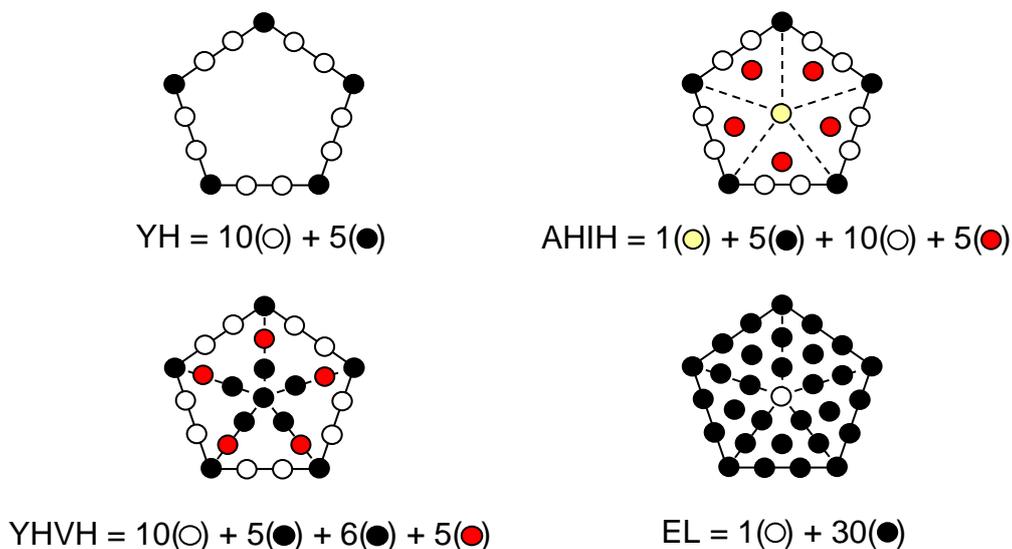


Figure 12. The various yod populations of the pentagon represent the letter values of the Godnames YAH, EHYEH, YAHWEH and EL.

with its enfolded polygons.

The pentagon is the polygonal representation of the Godnames of the four highest Sephiroth (Fig. 12). Constructed from tetractyses, it comprises **31** yods. This is the representation of EL, Godname of Chesed, because its central yod symbolises the letter value 1 of E (aleph) and the 30 yods surrounding its centre symbolise the letter

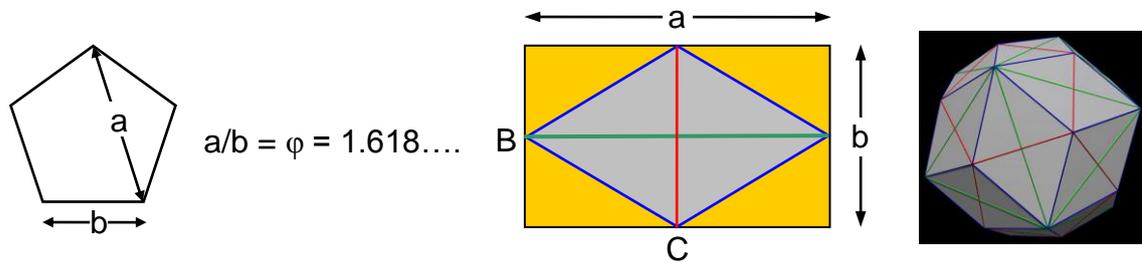


Figure 13. The ratio of the longer to shorter diagonals of the golden rhombic faces of the rhombic dodecahedron with B & C vertices is the Golden Ratio ϕ . This is the ratio of the lengths of a diagonal and a side of a pentagon.

value 30 of L (lamed). YAH is represented by the boundary of the pentagon because the letter value 10 of Y (yod) is the number of hexagonal yods between its five corners, which denote the number value 5 of H (heh). The number value **26** of YAHWEH is the number of radial and boundary yods. The number value **21** of EHYEH is the number of yods on the boundary of the pentagon, at its centre and at the centres of its five sectors.

It is well-known that the pentagon contains the Golden Proportions as the ratio of the lengths of its diagonal and side (Fig. 13). The very regular polygon whose proportions define the shape of the faces of the rhombic triacontahedron underlying the disdyakis triacontahedron embodies the Godname numbers **15** and **26** that characterise the numbers of sheets of B and C vertices defining the shape of these faces.

Listed below are the numbers of A, B and C vertices within each of the seven planes between two outermost A vertices:

	A	B	C	TOTAL	
	1				
1.	4	2	2	8	}
2.	4	2	4	10	
3.	4	0	2	6	
4.	4	4	4	12	}
5.	4	0	2	6	
6.	4	2	4	10	}
7.	4	2	2	8	
	1				

The fourth, central plane contains 12 vertices (six vertices and their images inverted through the centre of the disdyakis triacontahedron). There are $(1+8+10+6+6=31)$ independent vertices in the uppermost vertex and the four planes containing half of the polyhedron. This is how the Godname EL of Chesed with number value **31** prescribes its vertex population. The other **31** vertices are their mirror images. The total number of vertices in the four planes = $8 + 10 + 6 + 12 = 36$. This shows how the Godname ELOHA of the *next* Sephirah with number value **36** prescribes the polyhedron.

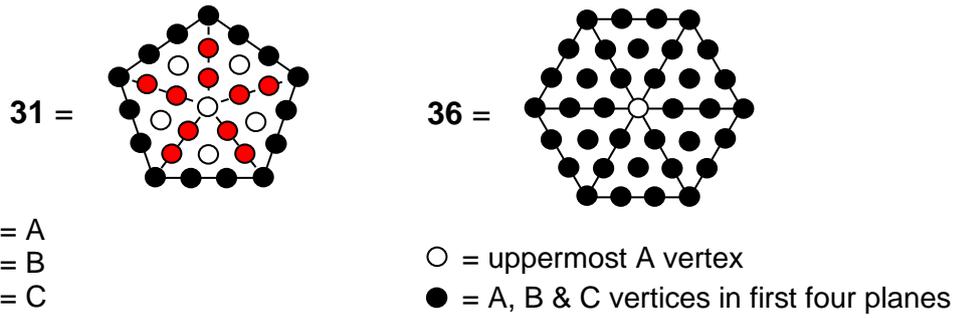


Figure 14. The **31** yods in the pentagon symbolise the **31** independent vertices of the disdyakis triacontahedron. The **37** yods in the hexagon symbolise the **37** vertices making up the uppermost vertex and the first four planes.

The **31** vertices are made up of **15** A vertices, six B vertices and 10 C vertices. In a pentagon divided into five tetractyses, there are **15** yods on its boundary, six yods at the centres of the tetractyses or the pentagon and 10 yods on the sides of tetractyses

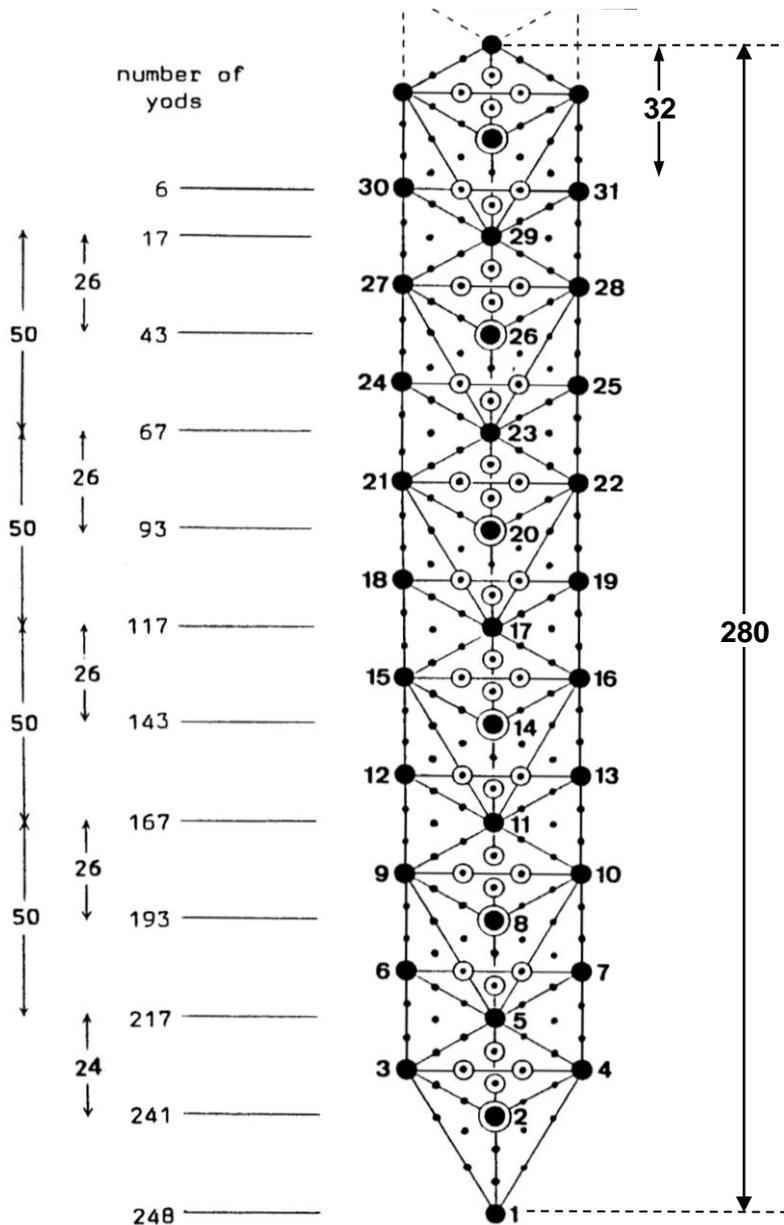


Figure 15. The number value **280** of *Sandalphon*, the Archangel of Malkuth, is the number of yods in the 5-tree with its triangles converted into tetractyses. There are **248** yods up to the **31st** SL, Chesed of the 5th tree. **248** is the number value of *Raziel*, the Archangel of Chokmah. EL, the Godname of Chesed with number value **31**, prescribes the dimension **248** of the superstring gauge symmetry group E_8 .

inside it (Fig. 14). Half of the polyhedron contains 37 vertices. This is the number of yods in a hexagon constructed from tetractyses, its central yod corresponding to the uppermost A vertex. The tetractys therefore reveals how these two polygons embody numbers defining the disdyakis triacontahedron as the inner form of the Tree of Life.

The number of vertices needed to create the disdyakis triacontahedron is not the number that actually constitutes it. The former is **31**; these vertices are symbolised by the **31** corners of the seven enfolded polygons unshared with the outer Tree of Life (see Fig. 8). The number constituting the first or last four planes is **36**; these are symbolised by all **36** corners of the seven enfolded polygons forming the inner Tree of Life.

n overlapping Trees of Life consist of $(12n+4)$ triangles. Transformed into tetractyses, they contain $(50n+20)$ yods, where **50** is the number value of ELOHIM. 10 Trees of Life have 520 yods in 124 tetractyses. The n-tree consists of $(12n+7)$ triangles. Changed into tetractyses, they contain $(50n+30)$ yods. The 5-tree has **280** yods (Fig. 15). This is the number value of *Sandalphon*, Archangel of Malkuth. As the apex of the 5-tree is Malkuth of the seventh tree, we see that Sandalphon prescribes the 7-tree representing the physical plane, the 25 tree levels of which are the 25 spatial dimensions predicted by quantum mechanics (hence the appropriateness in this context of the Sephirah Malkuth). The number value **26** of YAHWEH is the number of yods between successive SLs on the central pillar and the number value **50** of ELOHIM is the number of yods

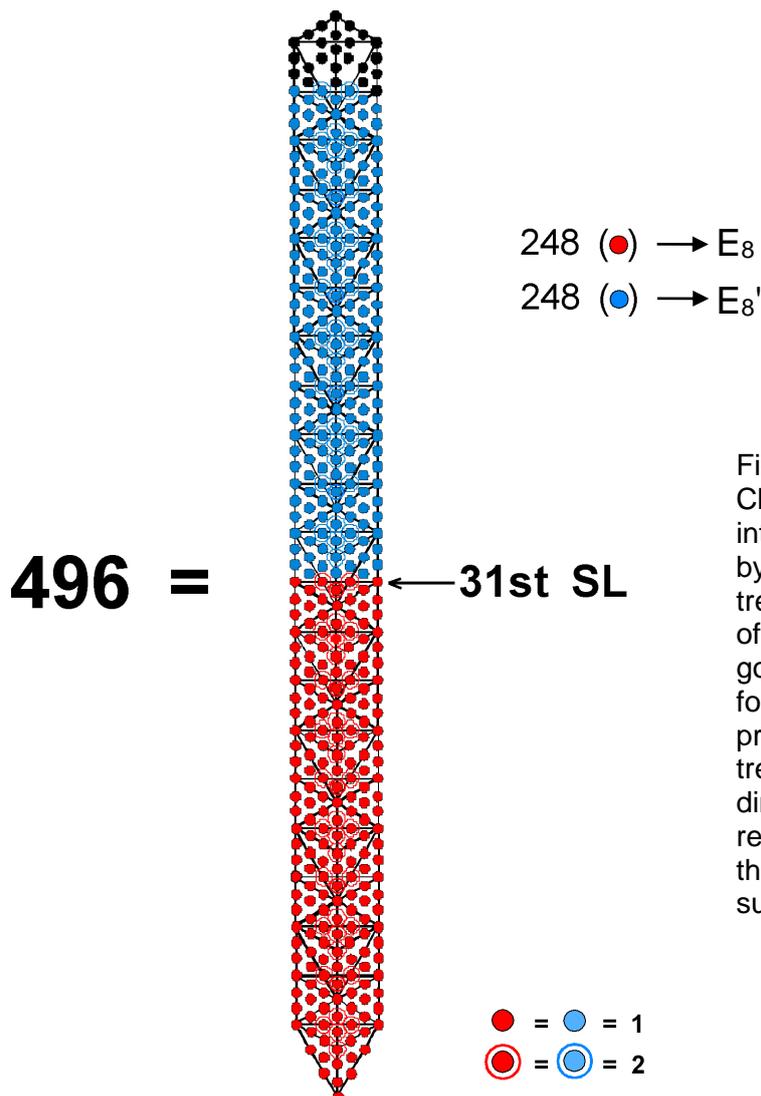


Figure 16. The **496** yods up to Chesed of the 10th tree are divided into two sets of **248** yods separated by the **31st** SL, Chesed of the 5th tree. This is the Tree of Life basis of the direct product $E_8 \times E_8'$ governing the heterotic superstring force. The Divine Name EL prescribes this direct product. The trees represent the 10 space-time dimensions of superstrings. Each red and blue yod denotes one of the $E_8 \times E_8'$ gauge fields transmitting superstring forces.

between corresponding SLs on this pillar. Chesed of the 5th tree — the first Sephirah of Construction — is the **31st** SL. Fig. 15 shows that there are **248** yods up to the level of the **31st** SL. This is how the Godname EL with number value **31** prescribes the dimension **248** of the superstring gauge symmetry group E_8 . Each yod up to the **31st** SL denotes a physical particle — a gauge boson of E_8 . The 1-tree contains **80** yods, where **80** is the number value of Yesod. There are, therefore, **(248–80=168)** yods above the 1-tree up to the **31st** SL. This is the number value of *Cholem Yesodoth*, (“Breaker of the Foundations”), the Mundane Chakra of Malkuth. As pointed out in Section 1 and as discussed in many previous articles, **168** is the structural parameter of the $E_8 \times E_8'$ heterotic superstring, being the number of circularly polarised oscillations of each of its whorls during one-half revolution.

The number of yods in n overlapping Trees of Life up to the level of Chesed of the n th tree = $50n - 3$.⁵ There are **496** yods up to (but not including) Chesed of the tenth tree (Fig. 16). They include 60 SLs, of which the lowest **31** SLs have **248** yods associated with them. Therefore, there are **248** yods above the **31st** SL up to Chesed of the tenth tree. The $E_8 \times E_8'$ heterotic superstring has a unified interaction transmitted by **496** gauge bosons, of which **248** particles are the gauge bosons of the first E_8 group and **248** particles are the gauge bosons of the second E_8' group. The Godname EL prescribes the direct product structure of the gauge symmetry group governing the interactions of heterotic superstrings. The product reflects the division of the ten Sephiroth of the Tree of Life into the five Sephiroth of its Lower Face and the five Sephiroth of its Upper Face.

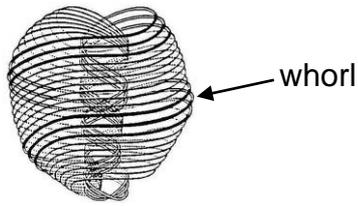
It was found in Article 22 that the disdyakis triacontahedron has 480 hexagonal yods in its 120 faces. This is the number of yods in the 9-tree, which has **31** tree levels, showing how the Divine Name EL prescribes the 480 hexagonal yods in the disdyakis triacontahedron that symbolise the 480 non-zero roots of $E_8 \times E_8'$.

There are **15** sheets of vertices orthogonal to a C-C axis and **26** sheets of vertices perpendicular to A-A & B-B axes, totalling 33 sheets. Against the backdrop of CTOL — the map of physical and superphysical reality — the **15-tree** has 780 yods, i.e., the yods in 78 tetractyses, and the **26-tree** has 1330 yods, i.e., the yods in 133 tetractyses. 78 is the dimension of E_6 , the rank-6 exceptional group, and 133 is the dimension of E_7 , the rank-7 exceptional group. The two numbers **15** and **26** that specify the number of sheets of vertices that create the shapes of the faces of the disdyakis triacontahedron define the dimensions of two exceptional subgroups of the exceptional group E_8 . The 33-tree has 1680 yods. As pointed out in Article 22, this is the number of hexagonal yods in the sequence of seven polyhedra that evolve into the perfect disdyakis triacontahedron:

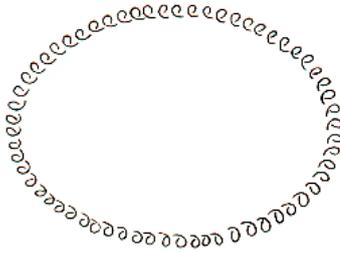
tetrahedron	48	} 240
octahedron	96	
cube	96	
icosahedron	240	
dodecahedron	240	
rhombic triacontahedron	240 + 240	
disdyakis triacontahedron	240 + 240	

TOTAL = 1680

It is the number of 1st-order spirillae in a whorl of the UPA. Here, therefore, is the scientific meaning of the numbers **15**, **26** and 33 defining the three types of groups of sheets of vertices. It is revealed by representing sheets by Trees of Life. Malkuth of the 33rd Tree of Life in CTOL is the **65th** SL on the central Pillar of Equilibrium. This is how



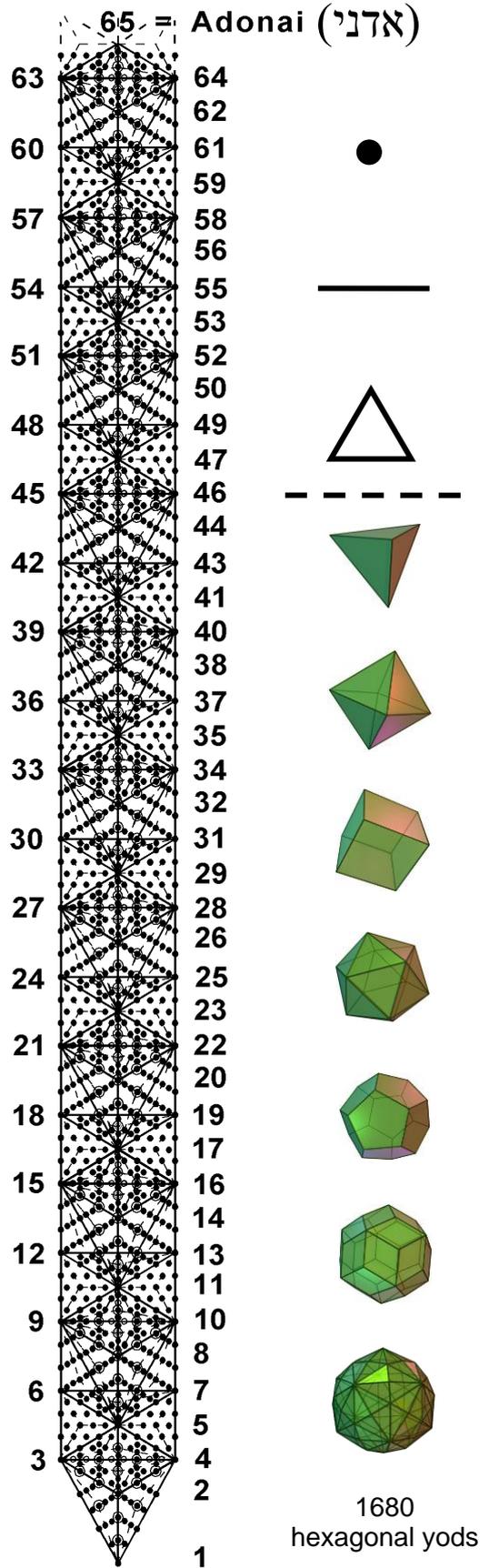
UPA/heterotic superstring
 (from: *Occult Chemistry*, Annie Besant & C.W. Leadbeater (1951))



The helical whorl
 has 1680 coils.

1680 =

Figure 17. The number of yods below the top (65th SL) of the tenth Tree of Life prescribed by ADONAI, the Godname assigned to Malkuth, is 1680. Each yod denotes a circularly polarised oscillation in the helical whorl of the $E_8 \times E_8$ heterotic superstring, described 111 years ago by Annie Besant and C.W. Leadbeater with a yogic siddhi called 'anima.' 1680 hexagonal yods are needed to build the seven polyhedra ending in the perfect disdyakis triacontahedron — the 3-d realisation of the inner Tree of Life.



triangles of the trees are divided into three tetractyses as there are in the 33-tree with its triangles turned into tetractyses and with **65** SLs on the central pillar up to Malkuth of the 33rd tree. The reason for this correspondence is that the geometry of the Tree of Life has a fractal-like quality of being characterised by the same set of parameters whenever it maps sections of CTOL that are counterparts of each other in a way that need not be discussed here.

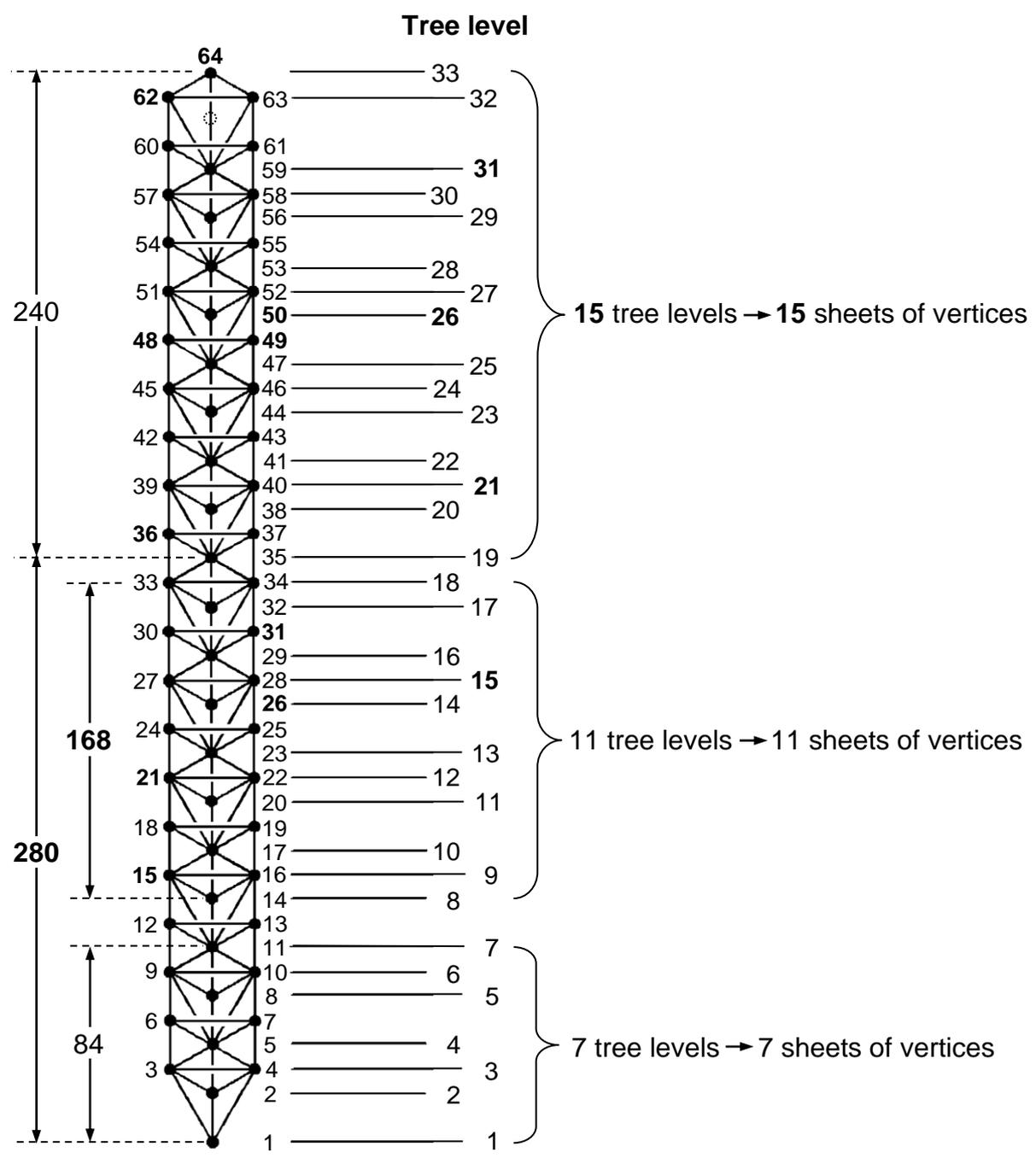


Figure 19. The correspondence between the 33 tree levels of 10 overlapping Trees of Life and the 33 sheets of vertices in the disdyakis triacontahedron. The 7 sheets of vertices correspond to the 7 tree levels of the 1-tree, which span 84 yods. The 11 sheets of vertices correspond to the next 11 tree levels; they span **168** yods. The **15** sheets of vertices orthogonal to a C-C axis correspond to the highest **15** tree levels, which span 240 yods.

We have seen that the 33 sheets of vertices in the disdyakis triacontahedron correspond to the 33 tree levels of 10 overlapping Trees of Life. The question now arises: how should the three types of groups of sheets be ordered when correlating them with tree levels? We know that the 1-tree has seven tree levels (Fig. 1), so it seems natural to correlate them with the 7 sheets of vertices perpendicular to an A-A axis. The 1-tree is the Malkuth level of 10 overlapping Trees of Life representing the 10 Sephiroth. The seven sheets of vertices represent the lowest tree and therefore the Malkuth aspect of the disdyakis triacontahedron as the polyhedral counterpart of these ten trees. There are **80** yods in the 1-tree and, as Fig. 15 indicates, there are four yods outside the 1-tree up to the level of its apex. The seven tree levels therefore span 84 yods (Fig. 19), where

$$84 = \begin{array}{ccc} 1^2 & & 3^2 \\ & \square & \\ 7^2 & & 5^2 \end{array}$$

is the sum of the squares of the first four odd integers. The 5-tree has **280** yods, of which seven yods are above the Binah-Chesed path of the 5th tree. This leaves 273

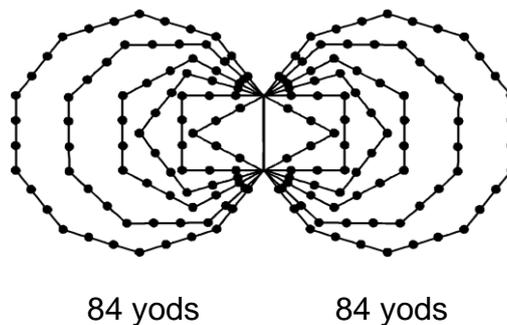


Figure 20. **168** yods lie outside the root edge on the boundaries of the pair of six regular polygons, 84 per set. They symbolise the **168** circularly polarised oscillations made in a half revolution of a whorl of the $E_8 \times E_8'$ heterotic superstring.

yods in the five trees up to the level of Chokmah of the 5th tree — the 18th tree level. Below Yesod of the 3rd tree are 105 yods (see Fig. 15). There are $(273 - 105 = 168)$ yods in the 11 tree levels from the 8th to the 18th. Finally, as the 10 overlapping Trees of Life have 520 yods, there are $(520 - 280 = 240)$ yods in the **15** tree levels between the 5-tree and the top of the 10th tree.

The superstring significance of the numbers 84, **168** and 240 is as follows: each whorl of the heterotic superstring twists five times around its central axis (see Fig. 17), making 1680 circularly polarised oscillations. There are $1680/5 = 336$ oscillations per revolution, **168** oscillations per half-revolution and 84 oscillations per quarter-revolution. The 84:84 pattern manifests in the inner form of the Tree of Life as the 84 yods that line the first six polygons outside the root edge on each side (Fig. 20). This pattern has been discussed in the context of octonions⁸ and the Klein Configuration.⁹ The seven sheets of vertices between diametrically opposite A vertices of the disdyakis triacontahedron define the 84 oscillations made by a whorl in a quarter of a revolution. The 11 sheets of vertices perpendicular to a B-B axis define the **168** oscillations in half a revolution. According to heterotic string theory, its E_8 gauge charges are spread around its length. However, it is the 240 charges corresponding to its non-zero roots that are spread

around the 10 whorls, 24 per whorl. There are **168** such charges along the seven minor whorls and **72** carried by the three major whorls. The former correspond to the **168** non-zero roots of E_8 that are not also non-zero roots of E_6 , one of its exceptional subgroups, whilst the latter correspond to the non-zero roots of E_6 . The **15** sheets of vertices perpendicular to a C-C axis correspond to the **15** tree levels spanning the 240 yods between the top of the 5-tree and the top of the 10th tree. These yods denote the gauge charges corresponding to the 240 non-zero roots of E_8 that are spread around the heterotic superstring. Information about the dynamics and oscillatory form of the heterotic superstring is thus encoded in the sheets containing the three types of vertices. The sheets of vertices perpendicular to an A-A axis and a B-B axis encode

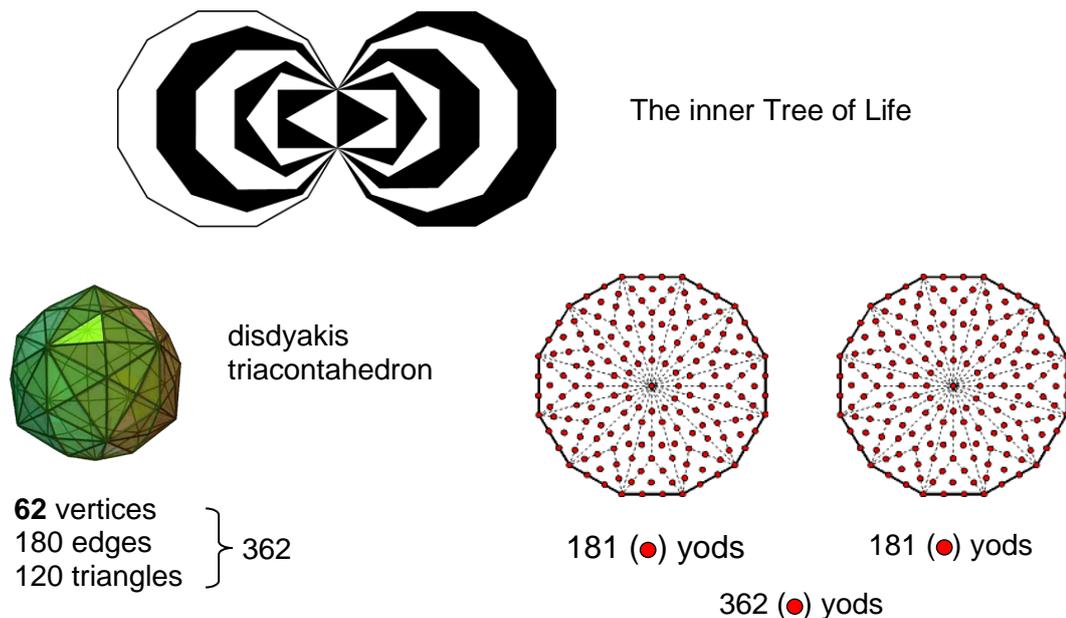
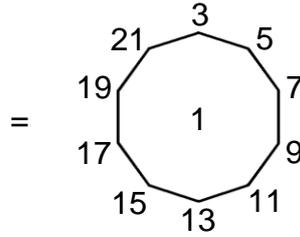


Figure 21. The 362 geometric elements composing the disdyakis triacontahedron are symbolised by the 362 yods in the pair of dodecagons — the last of the seven regular polygons constituting the inner Tree of Life.

through their Tree of Life counterparts the numbers of circularly polarised oscillations made in, respectively, a quarter-revolution and a half-revolution of a whorl, whilst the C-C sheets encode the number of E_8 gauge charges that are spread around all ten whorls.

The disdyakis triacontahedron is made up of **62** vertices, 180 edges and 120 triangles, that is, 362 geometric elements. This attribute is encoded in the pair of dodecagons — the last of the regular polygons making up the inner Tree of Life — because the division of their 24 sectors into three tetractyses produces 362 yods (Fig. 21). Each yod denotes a geometric element of the disdyakis triacontahedron. The two centres denote any pair of diametrically opposite A vertices and the 360 yods surrounding them denote the 60 vertices, 180 edges and 120 triangles between these opposite poles. The archetypal 'form' quality of the Godname ELOHIM assigned to Binah in the Tree of Life is illustrated by the fact that its number value **50** is the number of corners of the **72** tetractyses in the pair of dodecagons. The same applies to the Godname ELOHA assigned to Geburah below Binah in the Tree of Life because its number value is **36**, which is the number of tetractyses in a dodecagon. Each one has 25 corners and 60 edges, a total of 121 geometric elements, where $121 = 11^2$



“1” denotes the centre of the dodecagon. This shows how the decagon symbolising the perfect Pythagorean Decad expresses the tetractys-divided dodecagon.

As well as the Godname YAH assigned to Chokmah, the Godname EHYEH of Kether with number value **21** prescribes the superstring group-theoretical number 240. It is the sum of the first **21** binomial coefficients other than 1 in Pascal’s Triangle:

$$\begin{array}{cccc}
 & & 2 & \\
 & & 3 & 3 \\
 & & 4 & 6 & 4 \\
 240 = & 5 & 10 & 10 & 5 \\
 & 4 & 15 & 20 & 15 & 6 \\
 & 7 & 21 & 35 & 35 & 21 & 7 .
 \end{array}$$

As $24 = 5^2 - 1 = 3 + 5 + 7 + 9$, the number has the tetractys representation

$$\begin{array}{cccc}
 & & 3 & & 5 & & 7 & & 9 \\
 & & 3 & 3 & & 5 & 5 & & 7 & 7 & & 9 & 9 \\
 240 = & 3 & 3 & 3 & + & 5 & 5 & 5 & + & 7 & 7 & 7 & + & 9 & 9 & 9 \\
 & 3 & 3 & 3 & 3 & 5 & 5 & 5 & 5 & 7 & 7 & 7 & 7 & 9 & 9 & 9 & 9 .
 \end{array}$$

In terms of the integers 1, 2, 3 & 4 symbolised by the tetractys, $240 = (1+2+3+4)1 \times 2 \times 3 \times 4$, whilst 33 ($=1!+2!+3!+4!$) is the number of tree levels in 10 overlapping Trees of Life, the counterparts of which in the disdyakis triacontahedron are the 33 sheets of its three types of vertices.

These arithmetic expressions for geometrical parameters of holistic objects such as this polyhedron are examples of the Tetrad Principle at work.¹⁰ It also determines the $E_8 \times E_8$ heterotic superstring structural parameter 84 because

$$84 = 1^2 + 3^2 + 5^2 + 7^2$$

Indeed, the fact that the disdyakis triacontahedron has 120 vertices is a clear illustration of this fundamental mathematical principle because the 30 Golden Rhombic faces of the rhombic triacontahedron are each divided into four faces to produce 120 faces, where

$$30 = 1^2 + 2^2 + 3^2 + 4^2$$

and

$$\begin{aligned}
 120 &= 4 \times 30 = 2^2 \times (1^2 + 2^2 + 3^2 + 4^2) \\
 &= 2^2 + 4^2 + 6^2 + 8^2.
 \end{aligned}$$

As **248** = 4x**62**, the disdyakis triacontahedron with the Tetrad assigned to each of its **62** vertices represents the **248** E_8 gauge fields that transmit superstring forces, two diametrically opposite A vertices signifying the eight gauge fields that correspond to the eight zero roots of E_8 and the 60 vertices in the seven sheets between them generating the number 240 as the 240 non-zero roots and so signifying their corresponding 240

Table 2. The number values of the ten Sephiroth in the four Worlds.

	SEPHIRAH	GODNAME	ARCHANGEL	ORDER OF ANGELS	MUNDANE CHAKRA
1	Kether (Crown) 620	EHYEH (I am) 21	Metatron (Angel of the Presence) 314	Chaioth ha Qadesh (Holy Living Creatures) 833	Rashith ha Gilgalim First Swirlings. (Primum Mobile) 636
2	Chokmah (Wisdom) 73	YAHWEH, YAH (The Lord) 26, 15	Raziel (Herald of the Deity) 248	Auphanim (Wheels) 187	Masloth (The Sphere of the Zodiac) 140
3	Binah (Understanding) 67	ELOHIM (God in multiplicity) 50	Tzaphkiel (Contemplation of God) 311	Aralim (Thrones) 282	Shabathai Rest. (Saturn) 317
	Daath (Knowledge) 474				
4	Chesed (Mercy) 72	EL (God) 31	Tzadkiel (Benevolence of God) 62	Chasmalim (Shining Ones) 428	Tzadekh Righteousness. (Jupiter) 194
5	Geburah (Severity) 216	ELOHA (The Almighty) 36	Samael (Severity of God) 131	Seraphim (Fiery Serpents) 630	Madim Vehement Strength. (Mars) 95
6	Tiphareth (Beauty) 1081	YAHWEH ELOHIM (God the Creator) 76	Michael (Like unto God) 101	Malachim (Kings) 140	Shemesh The Solar Light. (Sun) 640
7	Netzach (Victory) 148	YAHWEH SABAOTH (Lord of Hosts) 129	Haniel (Grace of God) 97	Tarshishim or Elohim 1260	Nogah Glittering Splendour. (Venus) 64
8	Hod (Glory) 15	ELOHIM SABAOTH (God of Hosts) 153	Raphael (Divine Physician) 311	Beni Elohim (Sons of God) 112	Kokab The Stellar Light. (Mercury) 48
9	Yesod (Foundation) 80	SHADDAI EL CHAI (Almighty Living God) 49, 363	Gabriel (Strong Man of God) 246	Cherubim (The Strong) 272	Levanah The Lunar Flame. (Moon) 87
10	Malkuth (Kingdom) 496	ADONAI MELEKH (The Lord and King) 65, 155	Sandalphon (Manifest Messiah) 280	Ashim (Souls of Fire) 351	Cholem Yesodoth The Breaker of the Foundations. The Elements. (Earth) 168

The Sephiroth exist in the four Worlds of Atziluth, Beriah, Yetzirah and Assiyah. Corresponding to them are the Godnames, Archangels, Order of Angels and Mundane Chakras (their physical manifestation). This table gives their number values obtained by the ancient practice of gematria, wherein a number is assigned to each letter of the alphabet, thereby giving a number value to a word that is the sum of the numbers of its letters.

gauge fields. Constructed from tetractyses, the disdyakis triacontahedron has 422 yods on the 180 edges of the 120 tetractyses,¹¹ that is, 420 yods between polar-opposite A vertices are needed to define the edges of these tetractyses. Assigning the number 4 to these yods generates the number 1680. This is the number of oscillations in each whorl of the heterotic superstring. It is also the number of hexagonal yods symbolising the seven Sephiroth of Construction needed to build the sequence of seven polyhedra *completed* with the disdyakis triacontahedron. This is a profound meaning of the disdyakis triacontahedron: if we imagine it built with the integer 4 as each vertex, its seven sheets of vertices express both the number of particles transmitting the unified superstring force and the number of circularly polarised oscillations in each whorl of the heterotic superstring. Moreover, the *arithmetic* connection between 240 as the sum of the integers 4 assigned to the 60 vertices between opposite A vertices and 1680 — the sum of these integers assigned to the yods between them — is $1680 = 7 \times 240$. Remarkably, this factorisation is identical to the seven sets of 240 hexagonal yods in the sequence of seven polyhedra (see page 17). The disdyakis triacontahedron embodies numbers that characterise both fundamental *force* and *form*. It does so simply because, as shown in Articles 22-24 and as further demonstrated here, it is the 3-dimensional version of the universal blueprint called the ‘Tree of Life.’ No wonder that the ancient Pythagoreans gave to the number 4 the title of “holding the key of nature”!

References

- ¹ All numbers written in boldface are the number values of the ten Sephiroth, their Godnames, Archangels, Orders of Angels and Mundane Chakras. They are listed above in Table 2.
- ² Phillips, Stephen M. “ESP of Quarks & Superstrings,” New Age International, New Delhi, India, 1999.
- ³ Horava, Petr and Witten, Edward. “Heterotic and type 1 string dynamics from eleven dimensions,” *Nucl. Phys. B*460 (1996), pp. 506-524.
- ⁴ For the definition of the 2nd-order tetractys, see p. 4 of Article 24: “More Evidence for the Disdyakis Triacontahedron as the 3-dimensional Realisation of the Inner Tree of Life & its Manifestation in the $E_8 \times E_8$ Heterotic Superstring,” by Stephen M. Phillips, <http://www.smphillips.mysite.com/article24.pdf>.
- ⁵ The n-tree has $(50n+30)$ yods. Therefore, the number of yods in the $(n-1)$ -tree = $50(n-1) + 30 = 50n - 20$. There are 17 yods beyond the $(n-1)$ -tree up to the level of Chesed of the nth tree. The number of yods in n overlapping tree up to Chesed of the nth tree = $50n - 20 + 17 = 50n - 3$.
- ⁶ Phillips, Stephen M. Article 22: “The Disdyakis Triacontahedron as the 3-dimensional Counterpart of the Inner Tree of Life,” <http://www.smphillips.mysite.com/article22.pdf>, ref. 5.
- ⁷ Ibid, p. 16.
- ⁸ Phillips, Stephen M. Article 15: “The Mathematical Connection Between Superstrings and Their Micropsi Description: a Pointer Towards M-theory,” <http://www.smphillips.mysite.com/article15.pdf>, p. 11.
- ⁹ Ibid, p. 27, and Phillips, Stephen M. Article 21: “Isomorphism Between the I Ching table, the $3 \times 3 \times 3$ Array of Cubes and the Klein Configuration,” <http://www.smphillips.mysite.com/article21.pdf>, p. 6.
- ¹⁰ Phillips, Stephen M. Article 1: “The Pythagorean Nature of Superstring and Bosonic String Theories,” <http://www.smphillips.mysite.com/article01.pdf>, p. 5.
- ¹¹ Ref. 6, p. 9.