An interface between science and the transcendental

The 496 hexagonal yods in the outer & inner Trees of Life denote the 496 roots of $E_8 \times E_8$
The 31 SLs up to Chesed of the 5th Tree correspond to the 31 hexagonal yods either in each half of the outer Tree of Life or on the root edge. The 217 hexagonal yods up to this SL correspond to the 217 hexagonal yods intrinsic to each half of the inner Tree of Life.

Correspondence between 31:217 divisions of yods in the 5-tree and the combined outer/inner Trees of Life.
A. The number of yods in the n-tree is \( Y(n) = 50n + 30 \). Counting from the top of any Tree, there are 32 yods down to, but excluding, the Path joining Chesed and Geburah (shown in the diagram by the 32 blue yods). Hence, the number of yods in the n-tree up to Chesed of the nth Tree is \( Y(n) = Y(n) - 32 = 50n - 2 \). The number of yods up to Chesed of the 5th Tree is \( Y(5) = 248 \). This is the dimension of the rank-8, exceptional Lie group \( E_8 \), which is part of one of the two symmetry groups describing superstring forces that physicists Gary Schwarz and Michael Green found in 1984 are free of quantum anomalies because the groups have the crucial dimension 496, namely \( E_8 \times E_8 \) and SO(32). Each yod up to, and including, Chesed of the 5th Tree denotes one of the 248 roots of \( E_8 \). The number of SLs up to Chesed of the nth Tree is \( 6n + 1 \). There are 31 SLs up to Chesed of the 5th Tree and \( (248-31=217) \) yods that are not SLs, whereas 217 is the \( 216 \)th integer after 1 and 216 is the number value of Geburah, the Sephirah that follows Chesed, whose Godname EL has the number value 31. The \( 31:217 \) division displayed by the geometry of the 5-tree when constructed from tetractyses mirrors the arithmetic properties of the number 248 revealed by the number 496 being a perfect number, that is, an integer that is the sum of its divisors:

\[
496 = 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 31 + 1 \times 31 + 2 \times 31 + 4 \times 31 + 8 \times 31 = 31 + (1+2+4+7) \times 31 + 8 \times 31 = 31 + 217 + 248 = 248 + 248.
\]

As the sum of the first 5 divisors:

\[1 + 2 + 4 + 8 + 16 = 31\]

the number 31 is the number of corners of tetractyses (denoted in the diagram by black yods), whilst the number 217, which is \( 7 \times 31 \), is the number of their hexagonal yods (denoted by red yods).

B. The \( (7+7) \) enfolded Type A polygons making up the inner Tree of Life have 444 hexagonal yods. Two hexagonal yods are in the root edge shared by both sets of 7 enfolded polygons, so that 222 hexagonal yods can be associated with each set. The two pairs of white hexagonal yods inside each hexagon on vertical sides of tetractyses coincide with hexagonal yods on the side pillars of the Tree of Life, when its 16 triangles are tetractyses. Four such yods are shared with the 222 hexagonal yods in each set of polygons, so that 218 hexagonal yods are intrinsic to each set. One of them lies on the root edge. The outer Tree of Life has 70 yods when its triangles are tetractyses. Ten of these are corners and 60 are hexagonal yods (shown as green yods).

C. Because the outer Tree of Life has mirror symmetry, its 60 hexagonal yods can be distributed into a left-hand set of 30 (coloured brown) and a right-hand, mirror-image set of 30 (coloured green) provided that those aligned with the central Pillar of Equilibrium are divided into two sets that can be associated with one or the other "half." One hexagonal yod (coloured brown) in the root edge can be associated with the left-hand set and one hexagonal yod (coloured green) in the root edge can be associated with the right-hand set. The total number of hexagonal yods in the combined outer and inner Trees of Life is \( (1+30) + 217 + (1+30) + 217 = 31 + 217 + 31 + 217 = 248 + 248 = 496 \). This is the dimension of \( E_8 \times E_8 \). We find that the hexagonal yod composition of each half of the outer and inner Trees of Life matches the arithmetic properties of 248 arising from its being the largest divisor of 496. It is implausible in the extreme that this, too, could be due to coincidence. Moreover, the division:

\[ 31 = 1 + 30 \]

created by one hexagonal yod belonging to half of the inner Tree of Life and 30 hexagonal yods belonging to half of the outer Tree matches the gematria number value of EL (חג): \( \text{אל} = 1 \) and \( \text{lamed} = 30 \). The 31 yods in a pentagon constructed from tetractyses is the polygonal counterpart of this (see #11). Roughly speaking, the inner form of a single Tree of Life encodes its 10-fold differentiation into 10 Trees, each Sephirah being mapped by a complete Tree of Life, so that each half of its inner form maps 5 Trees up to Chesed, the first Sephirah of Construction. Each half of the inner Tree of Life encodes half of this potential 10-fold expansion. In fact, there are 496 yods up to, but excluding, Chesed of the highest Tree in 10 overlapping Trees of Life (see #12). Alternatively, there are 496 yods above the lowest point of 10 overlapping Trees up to, and including, Chesed of the 10th Tree. This brings out clearly the mathematical meaning of the gematria number value 496 of Malkuth:

\[ \text{מלכות} = 496 \]
It is the number of yods, counting from the first Sephirah of Construction of the 10th Tree, that are needed to construct from tetractyses 10 overlapping Trees of Life before reaching their most Malkuth level, namely, the bottom of the lowest Tree. This number at the heart of superstring theory is still mysterious because physicists have no explanation for its origin; it just, so to speak, tumbles out of a few pages of algebra. Its Kabbalistic meaning may be metaphysical. But it has a logically coherent, mathematical basis that illuminates its remarkable presence in superstring theory as a cosmic parameter.

The number 217 is the number of hexagonal yods intrinsic to the 7 enfolded polygons that are associated with either set of polygons. It is also the number of yods lining the sides of their 47 tetractyses because the latter have 264 yods, of which 47 are hexagonal yods at their centres, leaving 217 yods on their boundaries:

Here are three ways in which this number parameterises the forms of each half of the inner Tree of Life and its outer Tree of Life counterpart, namely, 5 overlapping Trees of Life that represent the 5 Sephiroth in half the outer Tree of Life. As the topmost corner of the hexagon in a set of 7 enfolded polygons coincides with the lowest corner of the hexagon enfolded in the next higher Tree of Life, 216 boundary yods are intrinsic to the 7 polygons enfolded in successive Trees. 216 (=6^3) is the number value of Geburah, which is the 6th Sephirah of Construction, counting from Malkuth. The number value 36 (=6^2) of ELOHA, the Godname of Geburah, is the number of corners of the 7 enfolded polygons making up each half of the inner Tree of Life. This Godname also prescribes the 217 boundary yods of their tetractys sectors because a Type A 36-gon has 217 yods." Remarkably, the first 6 separate Type A polygons have 36 corners and 222 yods, which compares with the 36 corners and 222 hexagonal yods associated with each set of the 7 enfolded Type A polygons. This is an example of how certain numbers parameterising a holistic system re-appear in another system that is holistic, even if it is part of the first one.

A. The simple triangle embodies the number 217 because it is the number of yods the triangle contains when its sectors are 2nd-order tetractyses:

* 496 is the third perfect number. The first one is 6, which has divisors 1, 2 & 3, because 6 = 1 + 2 + 3; the second perfect number is 28, which has the divisors 1, 2, 4, 7 & 14, and 28 = 1 + 2 + 4 + 7 + 14.
** The number of yods in a Type A n-gon = 6n + 1 (see [here](#)). A 36-gon has 217 yods.
Its 31 tetractyses have 31 corners (black yods) and 186 hexagonal yods (red yods), so that 217 = 1 (centre) + 30 + 186. There are 62 hexagonal yods per sector, where 62 is the number value of Tzadkiel, the Archangel of Chesed, whose Godname EL has the number value 31. Therefore,

\[186 = 62 + 62 + 62 = 62 + 124,\]

so that \[217 = 31 + 62 + 124.\]

These are three consecutive divisors of the perfect number 496 (see above).

B. An octagon with 8 sectors has \((8 \times 62 = 496)\) hexagonal yods (248 red hexagonal yods in 4 sectors and 248 blue hexagonal yods in the other 4 sectors). We see that the direct product \((E_8 \times E_8)\) nature of one of the two anomaly-free symmetry groups with dimension 496 reflects the simple fact that 8 = 2\(\times\)4. The number of corners of 1st-order tetractyses surrounding the centre of the octagon is 80, which is the number of Yesod. This is how the octagon with 2nd-order tetractyses as its sectors embodies the gematria number values of the last two Sephiroth.

C. As a 2nd-order tetractys contains 85 yods, where

\[85 = 4^0 + 4^1 + 4^2 + 4^3,\]

an 8-fold array of 2nd-order tetractyses has \((8 \times 85 = 672)\) yods surrounding its shared centre. This is a representation of the holistic parameter 672, which is embodied in the inner Tree of Life as the 672 corners & sides surrounding the centres of the \((7 + 7)\) separate Type B polygons (see bottom of page here). It is also the geometrical counterpart of:
1. the 672 yods in the first 4 Platonic solids constructed from tetractyses (see \#15);
2. the 672 yods in the 3-d Sri Yantra other than the corners of its 42 Type B triangles (\#16);
3. the 672 corners, sides & triangles surrounding the centres of the two Type C dodecagons (\#17).

Its significance for \(E_8 \times E_8\) heterotic superstrings is discussed at length between \#11 and \#21 in 4-d sacred geometries/Polychorons and Gosset polytope, the 8-dimensional \(4_{21}\) polytope that represents the 240 roots of \(E_8\) having 6720 (\(=672\times10\)) edges. Remarkably, the octagon with 8 2nd-order tetractyses as its sectors embodies the dimension 496 of \(E_8 \times E_8\), whilst an 8-fold array of these 2nd-order tetractyses embodies (apart from the Pythagorean/Tree of Life factor of 10) the number of edges of the \(4_{21}\) polytope whose 240 vertices represent the 240 roots of \(E_8\).